

8 Aug 2017

Equation

variable

$$2x + (4) = (16) \text{ ans}$$

var  
coefficient

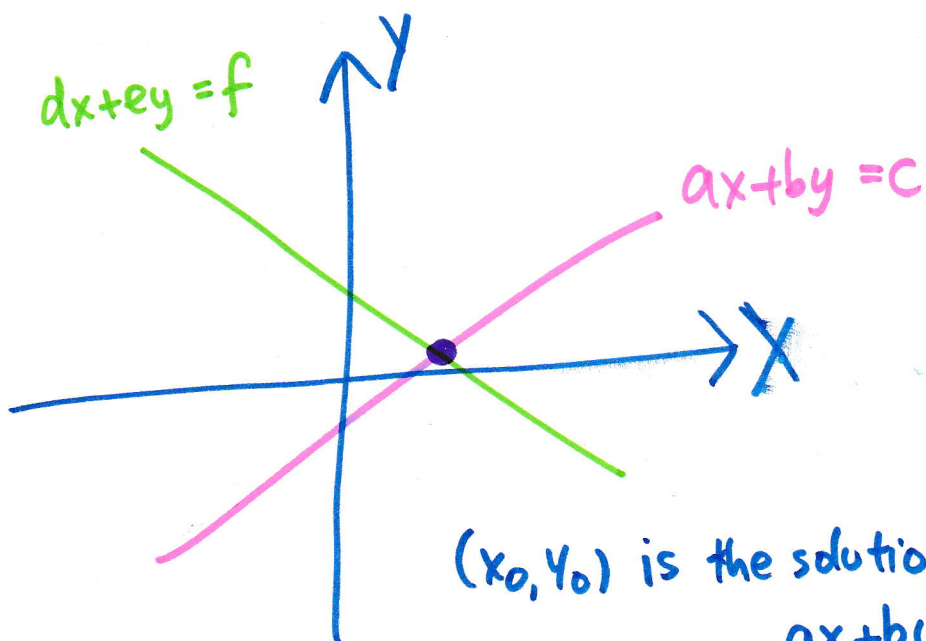
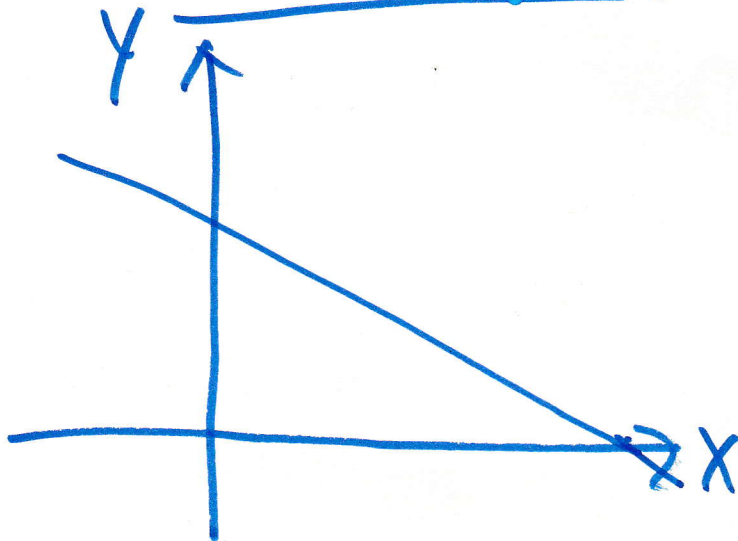
$$x = 6$$

Degree 1

linear equation

line

$$ax + by = c$$



$(x_0, y_0)$  is the solution of

$$ax + by = c \quad \text{--- (1)}$$

$$dx + ey = f \quad \text{--- (2)}$$

## Chapter 1: Matrices and Systems of Linear Equations

(Some contents of this note are taken from "Barnett/Ziegler/Byleen College Mathematics 12e" with modifications)

### Overview

1. To solve systems of linear equations in two variables by 3 methods

- Graphing
- Substitution
- Elimination by addition

2. Matrices

- Systems of linear equations and augmented matrices
- Elementary row operations and Gauss Jordan elimination
- Basic operations of matrices, inverse matrix and matrix equations

### Systems of two equations in two variables

We are given the linear system

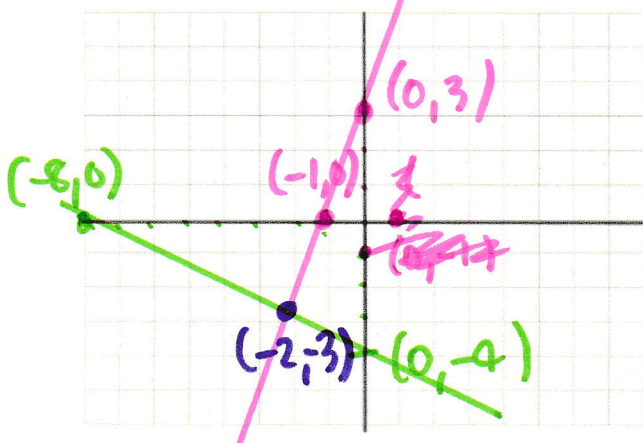
$$\begin{cases} ax + by = h \\ cx + dy = k \end{cases} \text{ system of linear equations}$$

A *solution* is an ordered pair  $(x_0, y_0)$  that will satisfy each equation. The solution set is the set of all ordered pairs that satisfy both equations.

Check that  $(-2, -3)$  is a solution of the system

$$\begin{cases} 3x - y = -3 \\ x + 2y = -8 \end{cases}$$

**Example 1** Solve the above system by graphing. (Note that you can only 'guess' the answer from where the two lines intersect.)



$$\begin{aligned} 3x - y &= -3 & \text{--- (1)} \\ x + 2y &= -8 & \text{--- (2)} \end{aligned}$$

partial x in  $y=0$   
 $x = -1$

partial y in  $x=0$   
 $y = 3$

partial x in  $y=0$   
 $x = -8$   $(-8, 0)$

y in  $x=0$   
 $y = -4$   $(0, -4)$

There is another method, which is 100% accurate, called the **method of substitution**.

Example 2 Solve the above system by substitution and compare the result

$$\begin{array}{lcl}
 3x - y = -3 & \text{--- (1)} & \\
 x + 2y = -8 & \text{--- (2)} & \\
 \text{mn (1)} & y = ? & \\
 & \boxed{y = 3x + 3} & \\
 \text{mnu } x = -2 & \text{nu } y = 3x + 3 & \text{nu } y = 3(-2) + 3 = -3 \\
 & \text{nu } (-2, -3) & 
 \end{array}$$

$$\begin{array}{rcl}
 \text{nu } y = 3x + 3 & \text{--- (2)} & \\
 x + 2(3x + 3) = -8 & & \\
 x + 6x + 6 = -8 & & \\
 7x + 6 = -8 & & \\
 7x = -14 & & \\
 \boxed{x = -2} & & 
 \end{array}$$

A better method is called **elimination by addition**. It can also generalize to larger systems. The following operations can be used to produce equivalent systems:

- Two equations can be interchanged.

$$\begin{array}{lcl}
 3x - y = -3 & \text{--- (1)} & \\
 x + 2y = -8 & \text{--- (2)} & \\
 \longleftrightarrow & & \\
 x + 2y = -8 & \text{--- (1')} & \\
 3x - y = -3 & \text{--- (2')} & 
 \end{array}$$

- An equation can be multiplied by a non-zero constant.

$$\begin{array}{lcl}
 \text{(1)} & 3x - y = -3 & \text{--- (1)} \\
 \text{nu (1) } \times 2 & \text{nu } 6x - 2y = -6 & \text{--- (3)}
 \end{array}$$

- A constant multiple of one equation can be added to another equation

$$\begin{array}{lcl}
 3x - y = -3 & \text{--- (1)} & \text{2x(1)} \rightarrow \\
 x + 2y = -8 & \text{--- (2)} & \\
 \hline
 6x - 2y = -6 & \text{--- (3)} & \\
 x + 2y = -8 & \text{--- (2)} & \\
 \hline
 7x + 0 = -14 & & 
 \end{array}$$

Example 3 Solve the above system using elimination by addition and compare the result.

$$\begin{array}{lcl}
 \boxed{x = -2} & & \\
 \text{nu } x = -2 & \text{nu } (2) & \\
 (-2) + 2(y) = -8 & & \\
 2y = -6 & & \\
 y = -3 & & 
 \end{array}$$