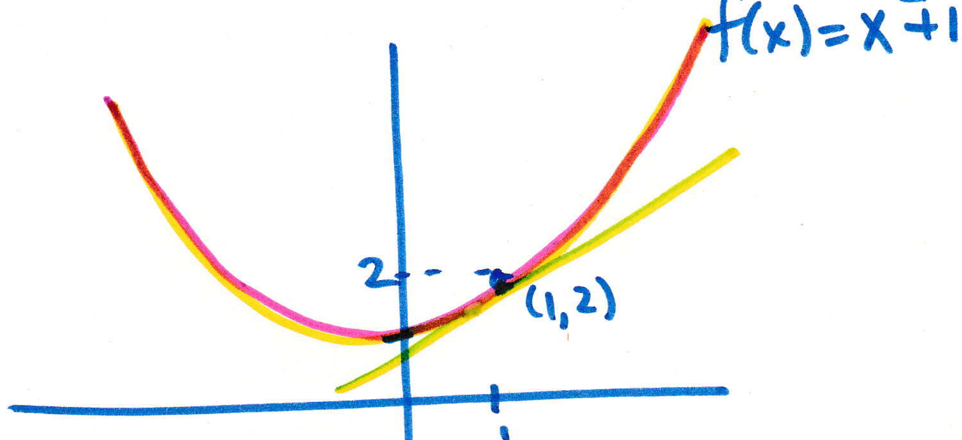
$$f(x) = 10x^2 \qquad \boxed{f'(x) = 20x}$$

$$f(x) = \frac{1}{x} \qquad \boxed{f'(x) = -\frac{1}{x^2}}$$

meaning ?



First Approach

$$m_{sec} = \frac{f(x) - f(x_0)}{x - x_0}$$

$$\boxed{m_{tan} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}}$$

Derivative

2.1 Tangent Lines and Rate of Change

In this section we will see how the concept of "tangent lines to a curve" and that of "the rate at which one variable changes relative to another" are related.

Tangent Lines

DEFINITION Suppose that x_0 is in the domain of the function f. The *tangent line* to the curve $y = f(x)$ at the point $P(x_0, f(x_0))$ is the line with equation

$$y - f(x_0) = m_{tan}(x - x_0)$$

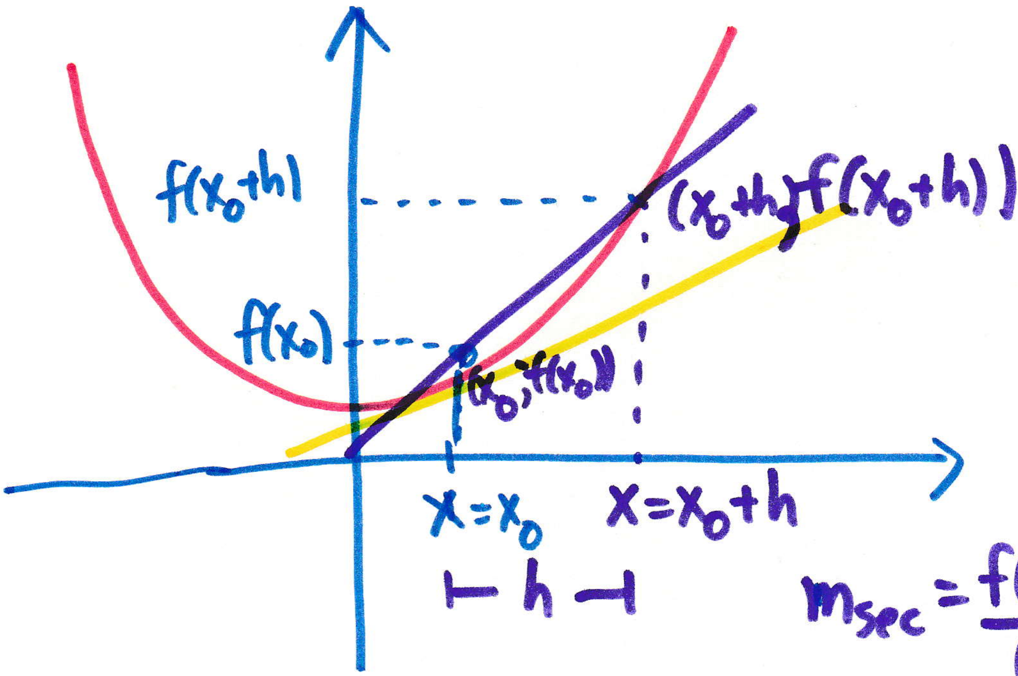
where

$$m_{tan} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \tag{2.1}$$

provided that the limit exists.

First Approach

Second Approach



$$m_{sec} = \frac{f(x_0+h)-f(x_0)}{(x_0+h)-x_0}$$

$$\boxed{m_{tan} = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}} = \frac{f(x_0+h)-f(x_0)}{h}$$

If we let $h = x - x_0$, then $x \to x_0$ is equivalent to $h \to 0$ and

$$m_{tan} = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} \tag{2.2}$$

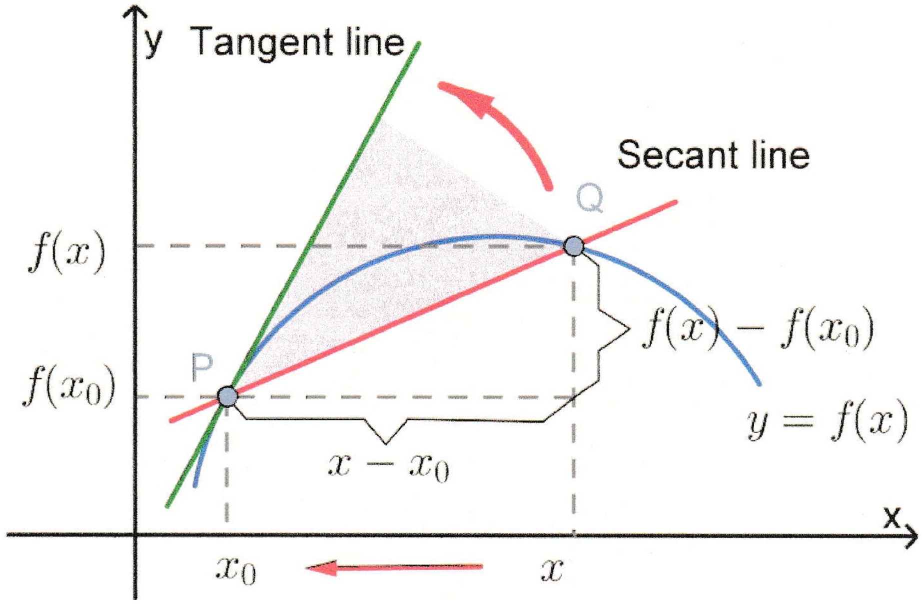


Figure 2.1: The slope of the tangent line as a limit of slopes of secant lines

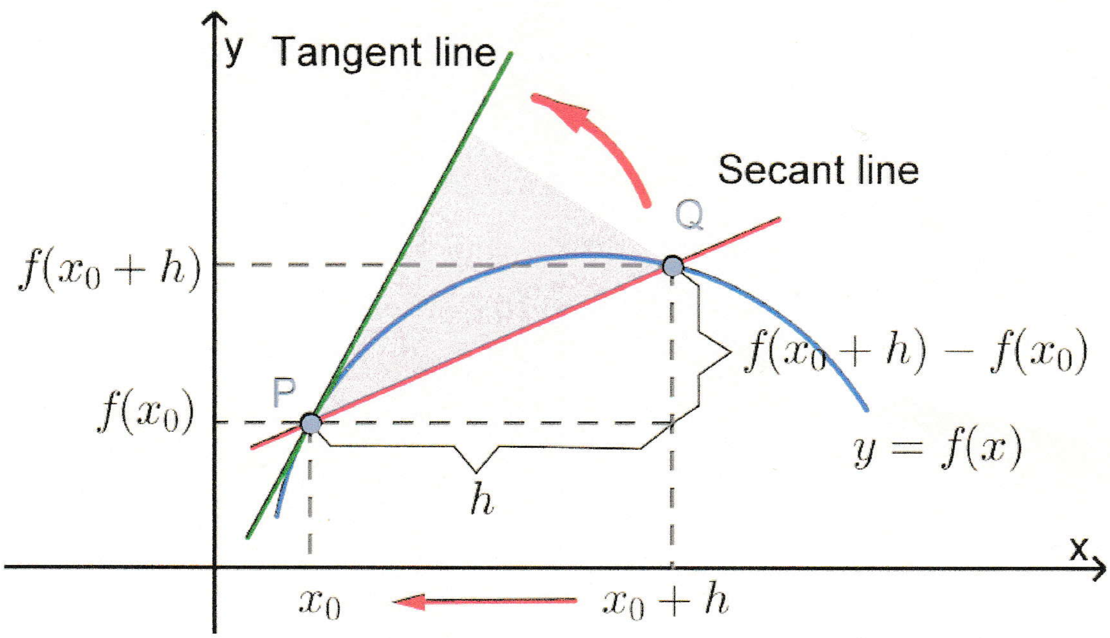
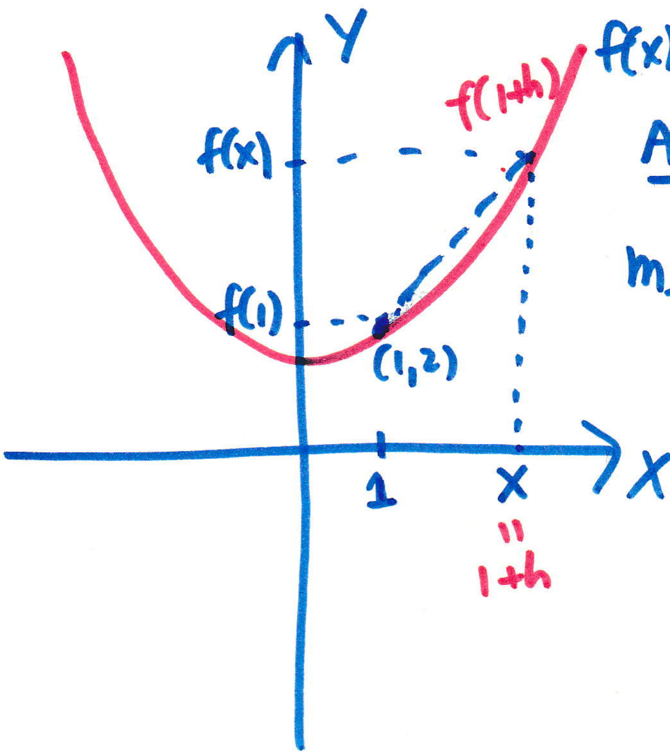


Figure 2.2: The slope of the tangent line as a limit of slopes of secant lines

Example 1 Use Formula (2.1) to find an equation for the tangent line to the parabola $y = x^2 + 1$ at the point $P(1, 2)$, and confirm the result agrees with that obtained in Example 1.

Example 2 Compute the slope in Example 1 using Formula (2.2).



Approach 1

$$m_{tan} = \lim_{x\to 1} \frac{f(x)-f(1)}{(x-1)}$$

$$= \lim_{x\to 1} \frac{[x^2+1]-2}{x-1}$$

$$= \lim_{x\to 1} \frac{x^2-1}{x-1}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$= 2 \qquad \#$$

ที่ $(x_0, y_0), m$

สมการ

$y - y_0 = m(x - x_0)$

∴ สมการเส้นสัมผัสกราฟที่ $(1,2)$ คือ

$$y - 2 = 2(x-1)$$

$$\boxed{y = 2x}$$

Approach 2.

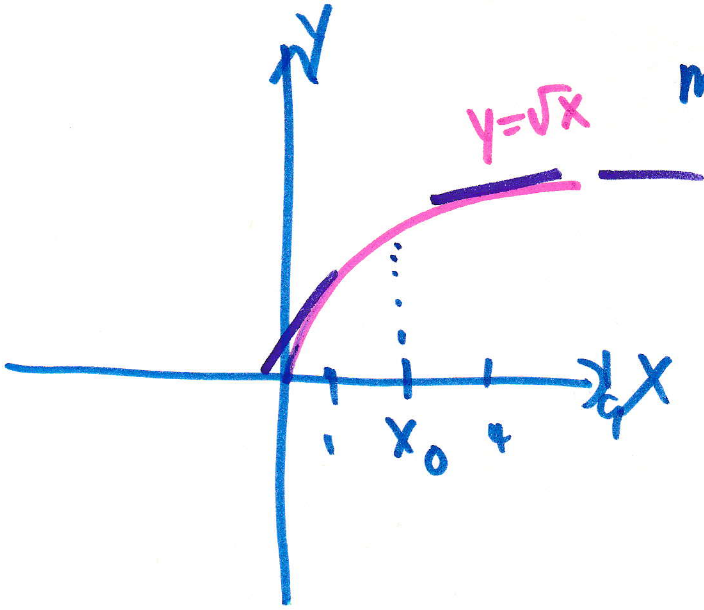
$$m_{tan} = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{[(1+h)^2+1]-2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1}+2\cancel{h}+h^2\cancel{+1}-\cancel{2}}{\cancel{h}}$$

$$= 2 \qquad \#$$

Example 3 Find the slopes of the tangent lines to the curve $y = \sqrt{x}$ at $x_0 = 1$, $x_0 = 4$, and $x_0 = 9$.



$$m_{tan} = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x_0+h} - \sqrt{x_0}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x_0+h} - \sqrt{x_0}}{h} \times \frac{\sqrt{x_0+h} + \sqrt{x_0}}{\sqrt{x_0+h} + \sqrt{x_0}}$$

$$= \lim_{h \to 0} \frac{(\cancel{x_0} + \cancel{h}) - \cancel{x_0}}{\cancel{h}(\sqrt{x_0+h} + \sqrt{x_0})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x_0+h} + \sqrt{x_0}}$$

$$= \frac{1}{2\sqrt{x_0}}$$

$\therefore$ เมื่อ $x_0 = 1$, $m_{tan} = \frac{1}{2}$

$x_0 = 4$, $m_{tan} = \frac{1}{4}$

$x_0 = 9$, $m_{tan} = \frac{1}{6}$

2.2 The Derivative Function

2.2.1 Definition of The Derivative Function

DEFINITION The function f' defined by the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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is called the *derivative of f with respect to x*. The domain of f' consists of all x in the domain of f where the limit exists.

The term "derivative" is used because the function f' is *derived* from the function f by a limiting process.

Example 6 Find the derivative with respect to x of $f(x) = x^2$, and use it to find an equation of the tangent line to $y = x^2$ at $x = 1$.

$f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2x\cancel{h} + h^2 - \cancel{x^2}}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h$$

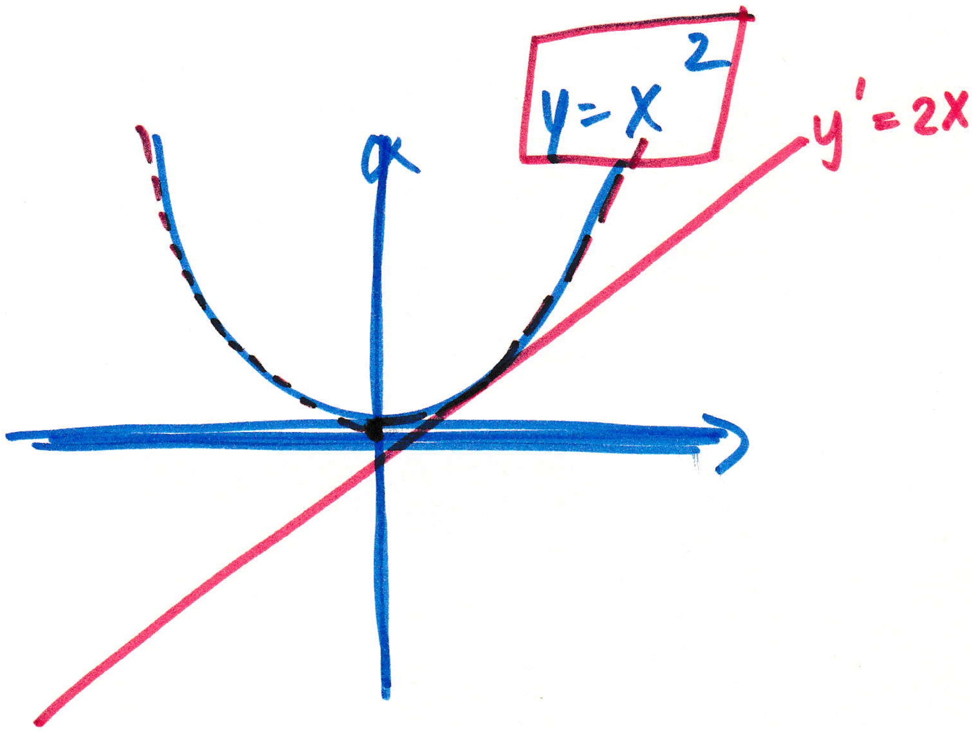
$$\boxed{f'(x) = 2x}$$

หาสมการเส้นสัมผัสโค้ง ของ $f(x) = x^2$ ที่ $x = 1$

ความชัน $f'(1) = 2$

ถ้า $x = 1, f(1) = 1$ $\therefore$ จุด $(1,1)$

$\therefore$ สมการเส้นสัมผัสโค้ง ที่ $(1,1)$, และมีความชัน $= 2$ คือ



y = x²
y' = 2x