4 Sep 2017

① $\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$

② $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

③ $\frac{d}{dx}(uv) = u\frac{du}{dx} + v\frac{du}{dx}$

$\frac{d}{dx}(\sqrt{u}) = \frac{1}{2\sqrt{u}}\frac{du}{dx}$

④ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

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⑤ $\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx}$

$\frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$

$\frac{d}{dx}(\tan u) = \sec^2 u\frac{du}{dx}$

$\frac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u\frac{du}{dx}$

$\frac{d}{dx}(\sec u) = \sec u \tan u\frac{du}{dx}$

$\frac{d}{dx}(\operatorname{cosec} u) = -\operatorname{cosec} u \cot u\frac{du}{dx}$

⑥ $\frac{d}{dx}(a^u) = a^u \ln a\frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$

$\frac{d}{dx}\log_a u = \frac{1}{u\ln a}\frac{du}{dx}$

$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$

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⑦ $\frac{d}{dx}(\sin^{-1}u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$\frac{d}{dx}(\tan^{-1}u) = \frac{1}{1+u^2}\frac{du}{dx}$

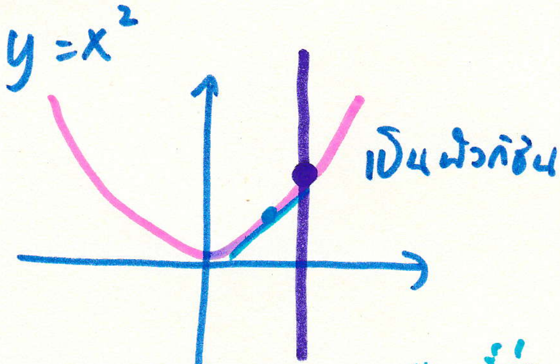
$\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+u^2}\frac{du}{dx}$

$\frac{d}{dx}(\sec^{-1}u) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

$\frac{d}{dx}(\operatorname{cosec}^{-1}x) = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

$f(x) = x^2 + 2x + 1$

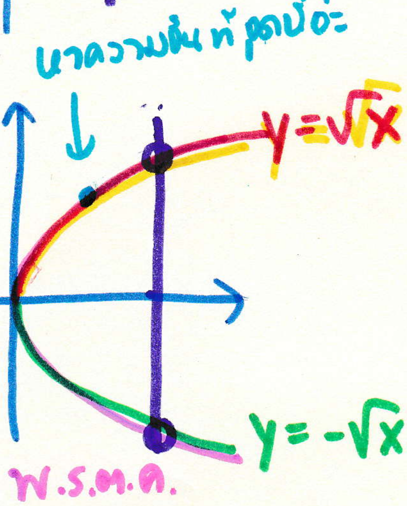
$f'(x) = 2x + 2$



$\boxed{y^2 = x}$

$y^2 = x$

$y = \pm\sqrt{x}$



$4xy^2 = 9$

$\Downarrow$

$y^2 = \frac{9}{4x} \Rightarrow y = \pm\sqrt{\frac{9}{4x}}$

$\boxed{4x^4y - y^3 = 4\sin y.}$

ฟังก์ชันโดยปริยาย (Implicit function)

$f(x) = 2x+2 \leftarrow$ ฟังก์ชันชัดแจ้ง explicit function.

(Sheet Page 51)

3.1 Implicit Function

Sometimes we deal with an expression that is not explicitly given but instead implicitly given in the form $f(x, y) = 0$ or $f(x, y) = g(x, y)$, e.g., $2y^2 = 3x + 8$. To find the derivative in this case,

1. first differentiate both sides of the expression,
2. then solve for $\frac{dy}{dx}$.

$y = \pm\sqrt{x-1}$

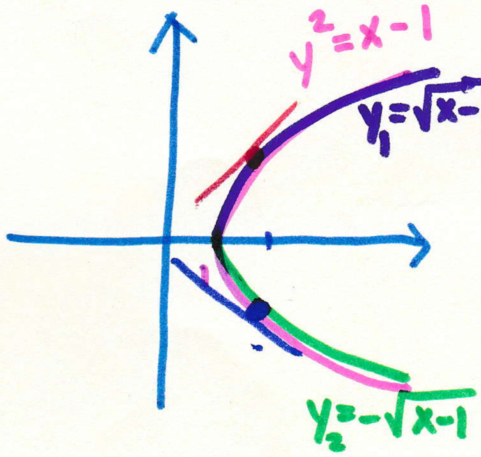
Example 1 Find $\frac{dy}{dx}$ where $y^2 - x + 1 = 0$. $\longrightarrow y^2 = x - 1$

diff ทั้งสองข้าง

$$\frac{d}{dx}(y^2 - x + 1) = \frac{d}{dx}(0)$$

$$\frac{dy^2}{dx} - \frac{dx}{dx} + \cancel{\frac{d(1)}{dx}} = \cancel{\frac{d(0)}{dx}}$$

$$2y\boxed{\frac{dy}{dx}} - 1 = 0$$



$$2y\frac{dy}{dx} = 1$$

$y_1 = \sqrt{x-1}$

$y_2 = -\sqrt{x-1}$

(1) $\boxed{\frac{dy}{dx} = \frac{1}{2y}}$ (?) $\Rightarrow$ $\boxed{\frac{dy_1}{dx} = \frac{1}{2\sqrt{x-1}}}$ ✓

(2) $\boxed{\frac{dy_2}{dx} = -\frac{1}{2\sqrt{x-1}}}$ ✓

หาความชันของเส้นสัมผัสโค้ง $y^2 - x + 1 = 0$
ที่จุด $(2, -1)$ และ $(2, 1)$

(1)

$$\left.\frac{dy}{dx}\right|_{(2,-1)} = \frac{1}{2(-1)} = -\frac{1}{2}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{1}{2(1)} = \frac{1}{2}$$

(2)

$$\left.\frac{dy}{dx}\right|_{(2,-1)} \Rightarrow \left.\frac{dy_2}{dx}\right|_{(2,-1)} = -\frac{1}{2}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} \Rightarrow \left.\frac{dy_1}{dx}\right|_{(2,1)} = \frac{1}{2}$$

Example 2 Find $\frac{dy}{dx}$ where $4xy^2 = 9$.

$$\frac{d}{dx}(4xy^2) = \frac{d}{dx}(9)$$

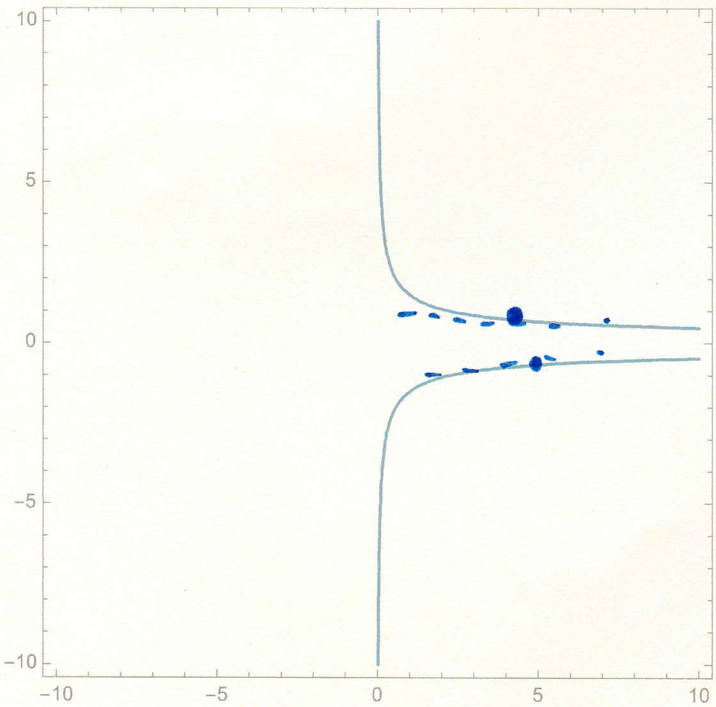
$$4\left[x\frac{dy^2}{dx} + y^2\frac{dx}{dx}\right] = \frac{d(9)}{dx}$$

$$4\left[(x)(2y)\frac{dy}{dx} + y^2\right] = 0$$

$$2xy\frac{dy}{dx} + y^2 = 0$$

$$2xy\boxed{\frac{dy}{dx}} = -y^2$$

$$\boxed{\frac{dy}{dx} = -\frac{y^2}{2xy}}$$



Example 3 Find $\frac{dy}{dx}$ where $4x^4y - y^3 = 4\sin y$.

$$\frac{d}{dx}(4x^4y - y^3) = \frac{d}{dx}(4\sin y)$$

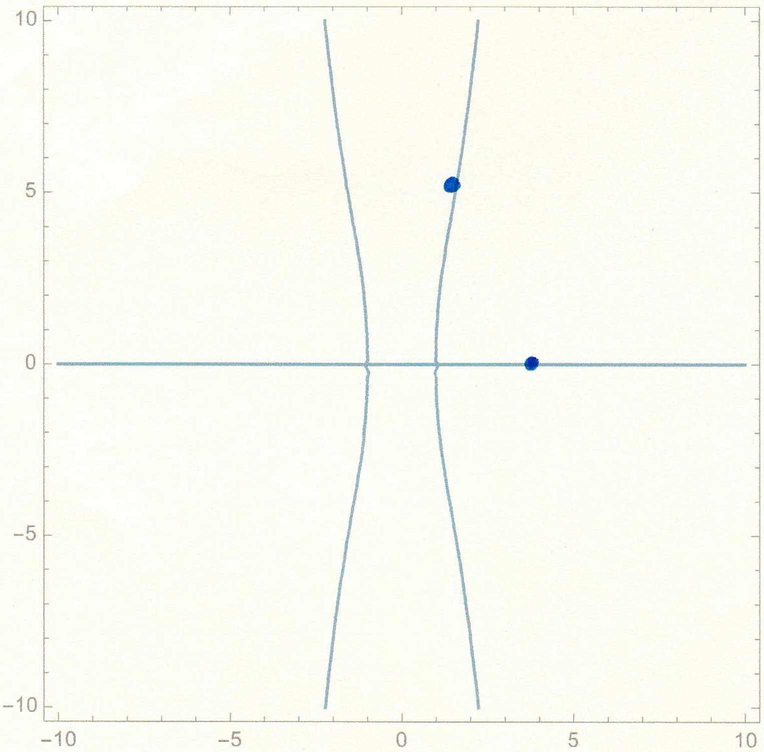
$$\frac{d}{dx}(4x^4y) - \frac{d}{dx}(y^3) = \frac{d}{dx}(4\sin y)$$

$$4\left(x^4\frac{dy}{dx} + y\frac{dx^4}{dx}\right) - 3y^2\frac{dy}{dx} = 4\cos y\frac{dy}{dx}$$

$$4x^4\boxed{\frac{dy}{dx}} + 16x^3y - 3y^2\boxed{\frac{dy}{dx}} = 4\cos y\boxed{\frac{dy}{dx}}$$

$$(4x^4 - 3y^2 - 4\cos y)\frac{dy}{dx} = -16x^3y$$

$$\boxed{\frac{dy}{dx} = -\frac{16x^3y}{4x^4 - 3y^2 - 4\cos y}}$$



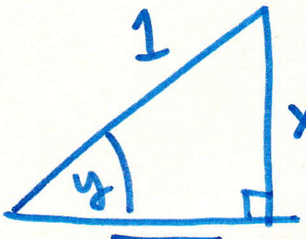
(Page 55)

Derive the formula of the inverse trigonometric function

THEOREM 3.2

1. $\dfrac{d}{dx}[\sin^{-1} u] = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$
2. $\dfrac{d}{dx}[\cos^{-1} u] = \dfrac{-1}{\sqrt{1-u^2}}\dfrac{du}{dx}$
3. $\dfrac{d}{dx}[\tan^{-1} u] = \dfrac{1}{1+u^2}\dfrac{du}{dx}$
4. $\dfrac{d}{dx}[\cot^{-1} u] = \dfrac{-1}{1+u^2}\dfrac{du}{dx}$
5. $\dfrac{d}{dx}[\sec^{-1} u] = \dfrac{1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}$
6. $\dfrac{d}{dx}[\csc^{-1} u] = \dfrac{-1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}$

Let $y = \sin^{-1} x$, then $x = \sin y$.



$\boxed{y = \sin^{-1} x}$

อยากได้ $\boxed{\dfrac{dy}{dx} = ?}$

ที่มุม y

$\cos y = \sqrt{1-x^2}$

$x = \sin y$

อยากได้ $\dfrac{dy}{dx}$

diff ทั้งสองข้าง

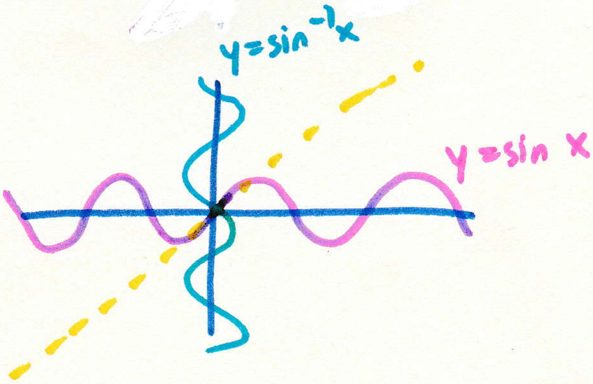
$$\frac{dx}{dx} = \frac{d}{dx}(\sin y)$$

$$1 = \cos y \boxed{\frac{dy}{dx}}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

จาก $\cos y = \sqrt{1-x^2}$

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$



HW* ถ้ารู้ว่า $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x\ln a}$ แล้วจะสามารถพิสูจน์สูตร $\dfrac{d}{dx}(a^x) = a^x \ln a$

Example 12 Find $\frac{dy}{dx}$ when $x^3 + x\tan^{-1}y = y$

$$\frac{d}{dx}(x^3 + x\tan^{-1}y) = \frac{dy}{dx}$$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(x\tan^{-1}y) = \frac{dy}{dx}$$

$$3x^2 + \left[x\frac{d}{dx}(\tan^{-1}y) + \tan^{-1}y\,\frac{dx}{dx}\right] = \frac{dy}{dx}$$

$$3x^2 + \left[x\left(\frac{1}{1+y^2}\right)\frac{dy}{dx} + \tan^{-1}y\right] = \frac{dy}{dx}$$

Hw

การบ้าน

การหอ

การคอนโด

การอพาร์ตเมนต์

(เฉลย)

$$\frac{x}{1+y^2}\frac{dy}{dx} - \frac{dy}{dx} = -3x^2 - \tan^{-1}y$$

$$\left(\frac{x}{1+y^2} - 1\right)\frac{dy}{dx} = -3x^2 - \tan^{-1}y$$

$$\therefore \frac{dy}{dx} = \frac{-3x^2 - \tan^{-1}y}{\left(\frac{x}{1+y^2} - 1\right)}$$

$$= \frac{(-3x^2 - \tan^{-1}y)(1+y^2)}{x - 1 - y^2} \quad \#$$

(Page 54)

3.2.2 Logarithm Differentiation

We now consider a technique called logarithmic differentiation. This technique is useful for functions that are composed of products, quotients, and powers.

Example 8 Find the derivative of $y = \dfrac{x \ln x \sqrt{x^2+1}}{(1+x^4)^2}$.

$\ln(a \cdot b) = \ln a + \ln b$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$\ln a^n = n \ln a$ ⊛

$\ln(a \pm b) \neq \ln a \pm \ln b$

$(\ln a)^n \neq n \ln a$

$\ln(a \cdot b) \neq \ln a \cdot \ln b$

อย่าทำ อันตราย!

$$\ln y = \ln\left(\frac{x(\ln x)\sqrt{x^2+1}}{(1+x^4)^2}\right)$$

$$\ln y = \ln x + \ln(\ln x) + \ln(\sqrt{x^2+1}) - \ln(1+x^4)^2$$

$$\ln y = \ln x + \ln(\ln x) + \underline{\ln(x^2+1)^{\frac{1}{2}}} - \underline{\ln(1+x^4)^2}$$

$$\ln y = \ln x + \ln(\ln x) + \frac{1}{2}\ln(x^2+1) - 2\ln(1+x^4)$$

(อยากได้ $\frac{dy}{dx}$)

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left[\ln x + \ln(\ln x) + \frac{1}{2}\ln(x^2+1) - 2\ln(1+x^4)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{\ln x}\frac{d(\ln x)}{dx} + \frac{1}{2}\cdot\frac{1}{x^2+1}\frac{d(x^2+1)}{dx} - 2\frac{1}{1+x^4}\frac{d(1+x^4)}{dx}$$

$$\frac{1}{y}\boxed{\frac{dy}{dx}} = \frac{1}{x} + \frac{1}{x\ln x} + \frac{\cancel{2}x}{\cancel{2}(x^2+1)} - \frac{2(4x^3)}{1+x^4}$$

$$\frac{dy}{dx} = y\left(\frac{1}{x} + \frac{1}{x\ln x} + \frac{x}{x^2+1} - \frac{8x^3}{1+x^4}\right) \quad \#$$

such that $y = \dfrac{x \ln x \sqrt{x^2+1}}{(1+x^4)^2}$.