เรื่อง อนุกรมฟังก์ชัน + quiz

① $f(x) = \tan^{99}(99x)$

$f'(x) = 99\tan^{98}(99x)\frac{d}{dx}(\tan 99x)$

$f(x) = \tan^{99}(99x)$ $\quad \tan^{99}(99x) = (\tan(99x))^{99}$

$f'(x) = \tan^{99}\frac{d}{dx}(99x) + (99x)\frac{d}{dx}(\tan^{99})$

$$f'(x) = 99\tan^{98}(99x)\sec^2(99x)\frac{d}{dx}(99x)$$

② $f(x) = \frac{1}{\sqrt{\sin x + \tan\sqrt{x}}}$ $\quad f(x) = \sqrt{\sin x + \tan\sqrt{x}}$

$$f'(x) = \frac{1}{2\sqrt{\sin x + \tan\sqrt{x}}}\frac{d}{dx}[\sin x + \tan\sqrt{x}]$$

$$= \frac{1}{2\sqrt{\sin x + \tan\sqrt{x}}}\left(\cos x + \sec^2\sqrt{x}\,\frac{d}{dx}(\sqrt{x})\right)$$

$$= \frac{1}{2\sqrt{\sin x + \tan\sqrt{x}}}\left(\cos x + \frac{\sec^2\sqrt{x}}{2\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{\sin x + \tan\sqrt{x}}}\left(\cos x + \sec^2\sqrt{x}\right)\frac{d\sqrt{x}}{dx}$$

③ $\frac{d}{dx}(2017^x) = 2017^x \ln(2017)$

$\frac{d}{dx}(2017^x) = x\,2017^{x-1}$

④ $\frac{d}{dx}(\pi^e) = 0$ $\quad \frac{d}{dx}(\pi^e) = e\pi^{e-1}$

$y = \sin x$ ↓

อินเวอร์สตรีโกน

$y = \arcsin x$

$y = \sin^{-1} x$

$y' = \dfrac{1}{\sqrt{1-x^2}}$

$y = \sin^{-1} x$ → $y = (\sin x)^{-1}$

$\dfrac{dy}{dx} = -1(\sin x)^{-2} \dfrac{d(\sin x)}{dx}$

ดัก $\sin^{-1} x$ $(\sin^{-1} x)^{-1}$

$= \dfrac{1}{\sin x}$

$= \operatorname{cosec} x$

$y = \tan^{-1}(x)$

$y' = \dfrac{1}{1+x^2}$

$\dfrac{d^2y}{dx^2}$

$y = \sin x$

หา $\dfrac{d^2y}{dx^2}$ → $\dfrac{dy}{dx} = \cos x$

$y = \sin x$

$\dfrac{d^2y}{dx^2} =$

$\left(\dfrac{dy}{dx}\right)^2$

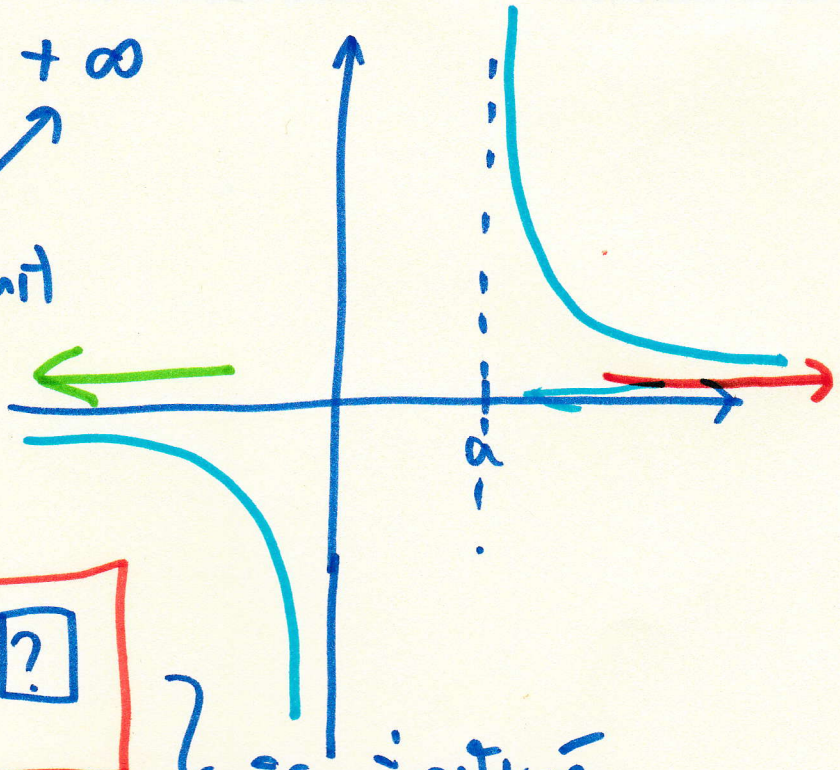
$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left[\dfrac{dy}{dx}\right]$

$= \dfrac{d}{dx}[\cos x]$

$= -\sin x$

$\lim_{x \to a^+} f(x) = +\infty$

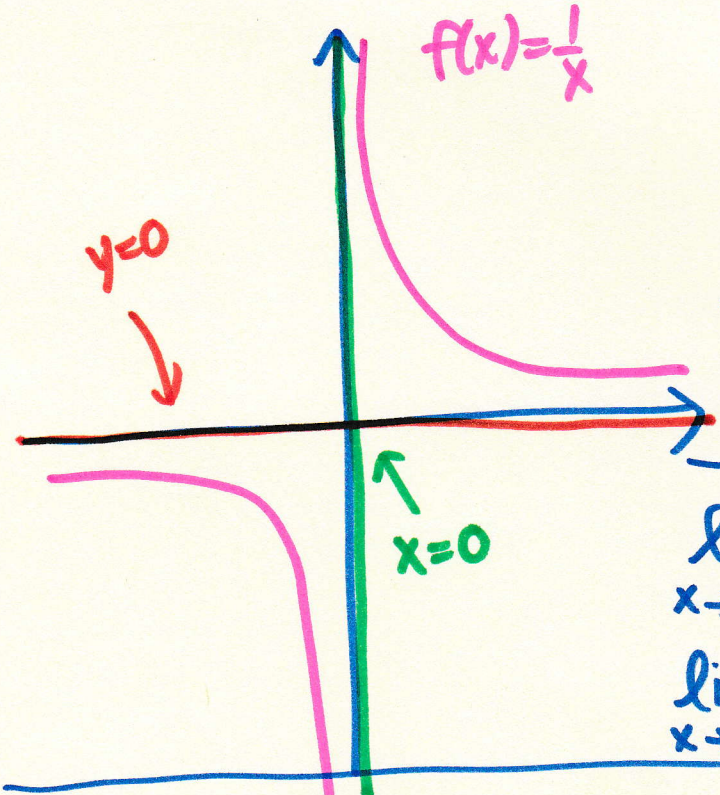
ลิมิตอนันต์
infinite limit



$\lim_{x \to \infty} f(x) = \boxed{?}$

$\lim_{x \to -\infty} f(x) = \boxed{?}$

} ลิมิตที่อนันต์
Limits at infinity.



$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = -\infty$

Vertical asymptote : $x = 0$
(เส้นตรงแนวตั้ง)

$\lim_{x \to +\infty} f(x) = \boxed{0}$ ← ค่าขอบเขต (bound)

$\lim_{x \to -\infty} f(x) = 0$

↑

$y = 0$ จะเป็น เส้นตรงแนวนอน (horizontal asymptote)

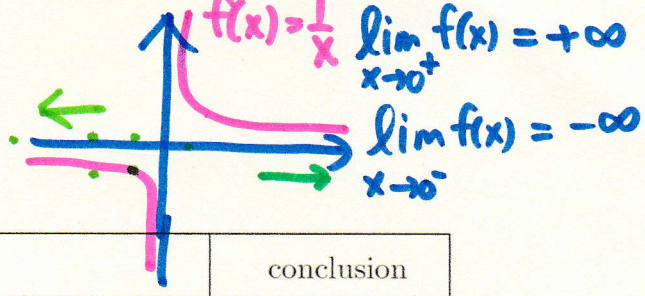
ถ้า $\lim_{x \to +\infty} f(x) = L$ แล้ว

$\lim_{x \to -\infty} f(x) = L$ แล้ว

$y = L$ จะเป็นเส้นตรงแนวนอน.

3.5 Limits at Infinity

In this section, we will discuss the behavior of functions as $x \to -\infty$ and $+\infty$ (the *end behavior* of the function).



Example 19 Consider the function $f(x) = \frac{1}{x}$.

As $x \to -\infty$

	values						conclusion
x	-1	-10	-100	-1000	-10000	⋯	as $x \to -\infty$
$\frac{1}{x}$	-1	-0.1	-0.01	-0.001	-0.0001	⋯	$\frac{1}{x} \to 0$

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$

As $x \to +\infty$

	values						conclusion
x	1	10	100	1000	10000	⋯	as $x \to +\infty$
$\frac{1}{x}$	1	0.1	0.01	0.001	0.0001	⋯	$\frac{1}{x} \to 0$

$$\lim_{x \to +\infty} \frac{1}{x} = 0$$

In general, if the values of $f(x)$ eventually get as close as we like to a number L as x decreases without bound. Then we write

$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad f(x) \to L \quad \text{as} \quad x \to -\infty.$$

Similarly, if the values of $f(x)$ eventually get as close as we like to a number L as x increases without bound. Then we write

$$\lim_{x \to +\infty} f(x) = L \quad \text{or} \quad f(x) \to L \quad \text{as} \quad x \to +\infty.$$

THEOREM 3.3 Let k, L, M be real numbers and f, g be functions such that $\lim_{x\to+\infty} f(x) = L$ and $\lim_{x\to+\infty} g(x) = M$. Then,

1. $\lim_{x\to+\infty} k = k$,
2. $\lim_{x\to+\infty} \frac{1}{x^a} = 0$, when a is positive,
3. $\lim_{x\to+\infty} [f(x) \pm g(x)] = L \pm M$,
4. $\lim_{x\to+\infty} f(x)g(x) = LM$,
5. $\lim_{x\to+\infty} \frac{f(x)}{g(x)} = \frac{L}{M}$, when $M \neq 0$,
6. $\lim_{x\to+\infty} [f(x)]^a = L^a$, when $a > 0$ and $L \geq 0$.

Note that the above theorem also holds in case of x approaching to $-\infty$.

Example 20 Find $\lim_{x\to-\infty} \left(\frac{5}{x} - 7\right)$.

$$\lim_{x\to-\infty} \left(\frac{5}{x} - 7\right) = \lim_{x\to-\infty} \frac{5}{x} - \lim_{x\to-\infty} 7$$

$$= 0 - 7$$

$$= -7$$

Infinite Limits at Infinity

if the values of $f(x)$ decrease or increase without bound as $x \to -\infty$, then we write

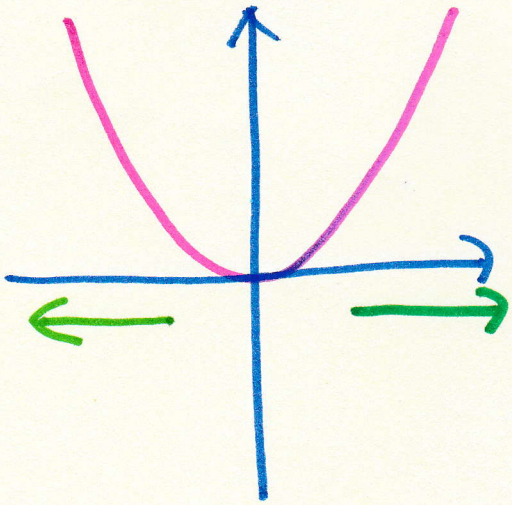
$$\lim_{x\to-\infty} f(x) = -\infty \text{ or } \lim_{x\to-\infty} f(x) = +\infty,$$

as appropriate. Similarly, if the values of $f(x)$ decrease or increase without bound as $x \to +\infty$, then we write

$$\lim_{x\to+\infty} f(x) = -\infty \text{ or } \lim_{x\to+\infty} f(x) = +\infty,$$

as appropriate.

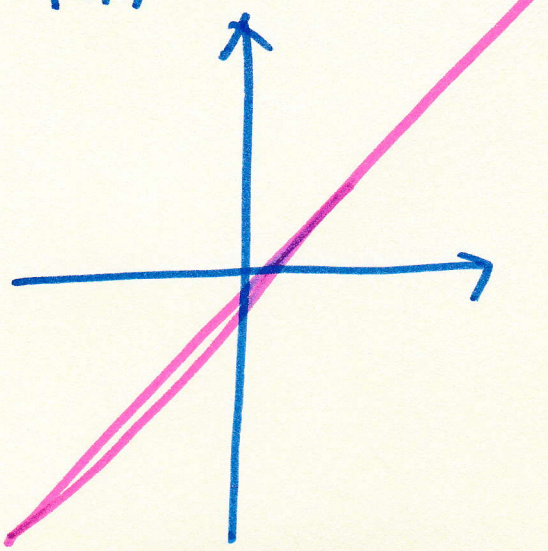
Ex $f(x) = x^2$



$\lim_{x\to+\infty} f(x) = +\infty$

$\lim_{x\to-\infty} f(x) = +\infty$

$f(x) = x$



$\lim_{x\to+\infty} f(x) = +\infty$

$\lim_{x\to-\infty} f(x) = -\infty$

Example 21 Find the following limits.

1. $\lim_{x\to-\infty} x =$ $-\infty$ $\quad\lim_{x\to+\infty} x =$ $+\infty$

2. $\lim_{x\to-\infty} x^2 =$ $+\infty$ $\quad\lim_{x\to+\infty} x^2 =$ $+\infty$

3. $\lim_{x\to-\infty} x^3 =$ $-\infty$ $\quad\lim_{x\to+\infty} x^3 =$ $+\infty$

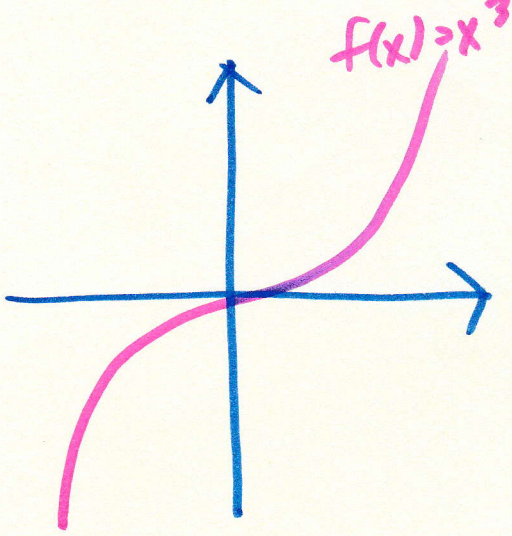
4. $\lim_{x\to-\infty} x^4 =$ $+\infty$ $\quad\lim_{x\to+\infty} x^4 =$ $+\infty$

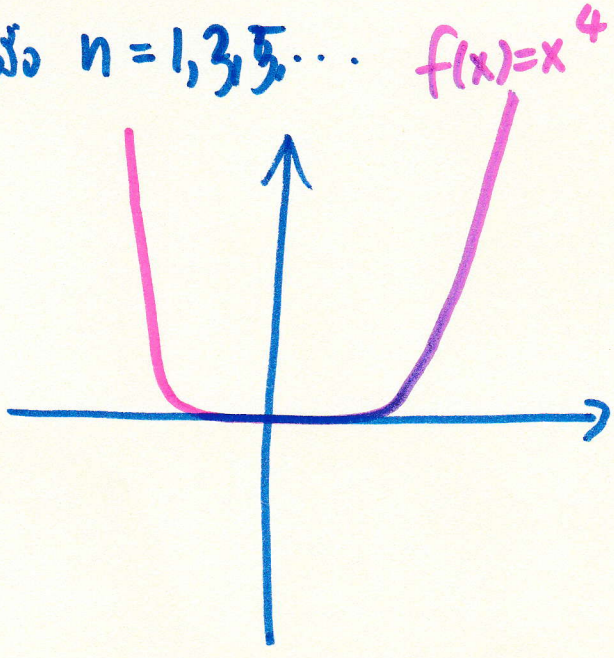
5. $\lim_{x\to-\infty} 15x =$ $-\infty$ $\quad\lim_{x\to+\infty} 15x =$ $+\infty$

6. $\lim_{x\to-\infty} -3x^2 =$ $-\infty$ $\quad\lim_{x\to+\infty} -3x^2 =$ $-\infty$

สรุป $\lim_{x\to+\infty} x^n = +\infty$ เมื่อ $n = 1, 2, 3, \ldots$

$$\lim_{x\to-\infty} x^n = \begin{cases} +\infty & \text{เมื่อ } n = 2, 4, 6, \ldots \\ -\infty & \text{เมื่อ } n = 1, 3, 5, \ldots \end{cases}$$





$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ } ฟังก์ชันพหุนาม polynomial function.

Ex $p(x) = x^2 + 2x + 4$

$+\infty \quad -\infty$

Ex 22.1 $\lim_{x \to -\infty} (5x^4 - 10x^3 + 897x + 84000)$ $\frac{\infty}{\infty}$

* รูปไม่กำหนด (indeterminate form)

$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty - \infty, 0^0, 1^\infty, \infty^0$

$= \lim_{x \to -\infty} x^4 \left(5 - \frac{10}{x} + \frac{897}{x^3} + \frac{84000}{x^4} \right)$ (0, 0, 0)

$= \lim_{x \to -\infty} 5x^4$

$= +\infty$

Ex 22.2 $\lim_{x \to +\infty} (-x^3 + 650x^2 + 7x - 1)$

พจน์ดีกรีสูงสุด $= \lim_{x \to +\infty} -x^3 = -\infty$

The end behavior of polynomials

The end behavior of a polynomial matches the end behavior of its highest degree term.

Example 22 Find the following limits.

1. $\lim_{x\to-\infty}(5x^4-10x^3+897x+84000) =$ $+\infty$

2. $\lim_{x\to+\infty}(-x^3+650x^2+7x-1) =$ $-\infty$

The end behavior of rational functions

One technique for determining the end behavior of a rational function is to divide each term in the numerator and denominator by the highest power of x that occurs in the denominator.

Example 23 Find the following limits.

1. $\lim_{x\to-\infty}\dfrac{24x-9}{8x+11}$. $= \lim_{x\to-\infty}\dfrac{\frac{24x}{x}-\frac{9}{x}}{\frac{8x}{x}+\frac{11}{x}} = \dfrac{24}{8} = 3$

2. $\lim_{x\to+\infty}\dfrac{3x^2-5x}{8x^6+14x-1}$. $= \lim_{x\to+\infty}\dfrac{\frac{3x^2}{x^6}-\frac{5x}{x^6}}{\frac{8x^6}{x^6}-\frac{14x}{x^6}-\frac{1}{x^6}} = \dfrac{0}{8} = 0$

Ex $\lim_{x\to+\infty}\dfrac{5x^3-2x^2+1}{1-3x} = \lim_{x\to+\infty}\dfrac{\frac{5x^3}{x}-\frac{2x^2}{x}+\frac{1}{x}}{\frac{1}{x}-\frac{3x}{x}} = -\infty$

End behavior of Trigonometric, Exponential and Logarithmic Functions

$$\left.\begin{array}{l}\lim\limits_{x\to-\infty}\sin x,\\ \lim\limits_{x\to+\infty}\sin x,\\ \lim\limits_{x\to-\infty}\cos x,\\ \lim\limits_{x\to+\infty}\cos x\end{array}\right\}$$ ไม่มีลิมิต .....

$\lim\limits_{x\to-\infty} e^x = 0$

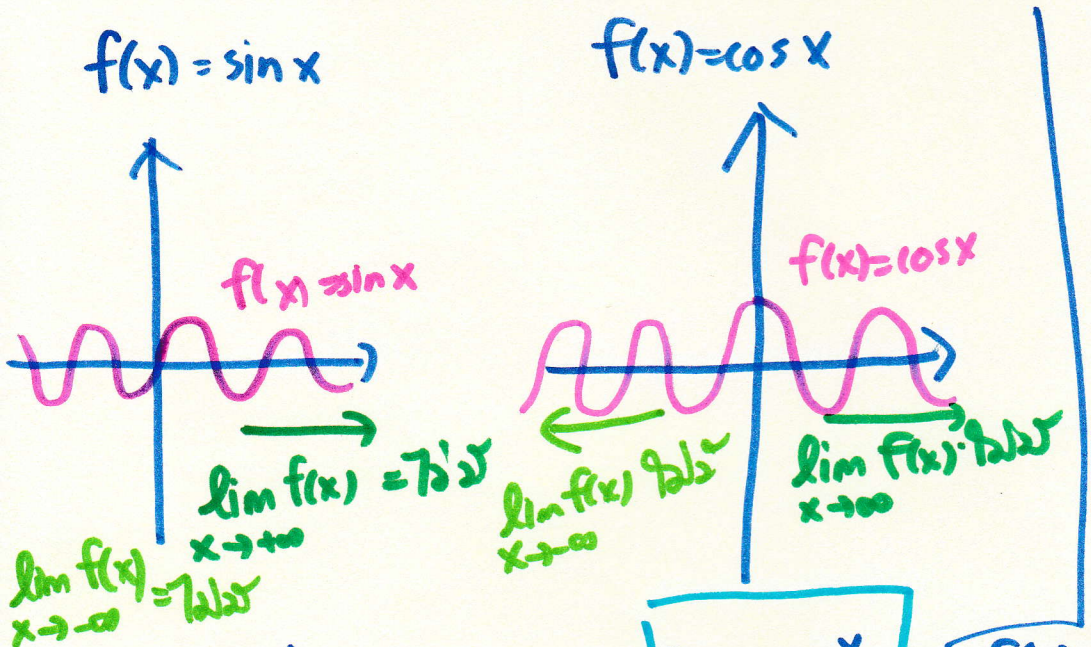
$\lim\limits_{x\to-\infty} e^{-x} = +\infty$

$\lim\limits_{x\to0^+} \ln x = -\infty$

$\lim\limits_{x\to+\infty} e^x = +\infty$

$\lim\limits_{x\to+\infty} e^{-x} = 0$

$\lim\limits_{x\to+\infty} \ln x = +\infty$



$f(x) = e^{-x} = \frac{1}{e^x}$