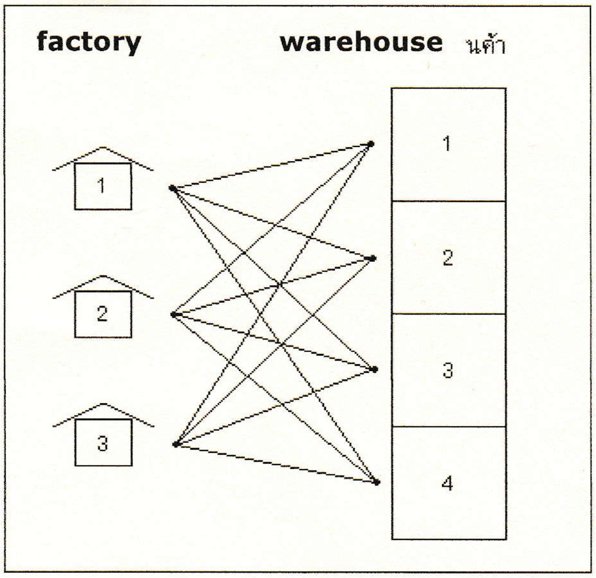
2.8 Transportation Problem



One of the first applications of linear programming was to the problem of minimizing the cost of transporting products or materials from several sources to several destinations.

Problems of this type are referred to as **transportation problems.**

To form this shipping schedule, we must decide how many products to ship from either factory to either warehouse. This will involve many decision variables:

The two objectives of transportation problems are

1. minimize the cost of shipping m units to n destinations or
2. maximize the profit of shipping m units to n destinations.

x_{ij} = number of products shipped from factory i to warehouse j

For example, x_{34} *= number of products shipped from factory 3 to warehouse 4*

a_i = number of products available from factory i
b_j = number of products required at warehouse j
c_{ij} = cost of shipping one unit of product from factory i to warehouse j.

$$\text{Min } C = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

s. t.

$$\begin{aligned}
x_{11} + x_{12} + x_{13} + x_{14} &\le a_1 \\
x_{21} + x_{22} + x_{23} + x_{24} &\le a_2 \\
x_{31} + x_{32} + x_{33} + x_{34} &\le a_3 \\
x_{11} + x_{21} + x_{31} &\ge b_1 \\
x_{12} + x_{22} + x_{32} &\ge b_2 \\
x_{13} + x_{23} + x_{33} &\ge b_3 \\
x_{14} + x_{24} + x_{34} &\ge b_4 \\
x_{ij} \ge 0 \ (i = 1, 2, 3 \text{ and } j &= 1, 2, 3, 4)
\end{aligned}$$

Example 2.8.1 A computer manufacturing company has two assembly plants, **plant A and plant B**, and two distribution outlets, **outlet I and outlet II**. Plant A can assemble at most 700 computers a month, and plant B can assemble at most 900 computers a month. Outlet I must have at least 500 computers a month, and outlet II must have at least 1,000 computers a month.

Transportation costs for shipping one computer from each plant to each outlet are as follows: \$6 from plant A to outlet I; \$5 from plant A to outlet II: \$4 from plant B to outlet I; \$8 from plant B to outlet II. Find a shipping schedule that will minimize the total cost of shipping the computers from the assembly plants to the distribution outlets. What is the minimum cost?

We define

x_1 = number of computers shipped from plant A to outlet I,
x_2 = number of computers shipped from plant A to outlet II,
x_3 = number of computers shipped from plant B to outlet I,
x_4 = number of computers shipped from plant B to outlet II.

We can create the LP problem with the following constraints.

$x_1 + x_2 \le 700$ (Available from A),

$x_3 + x_4 \le 900$ (Available from B),

$x_1 + x_3 \ge 500$ (Required at I),

$x_2 + x_4 \ge 1{,}000$ (Required at II).

So the total cost of shipping is: $C = 6x_1 + 5x_2 + 4x_3 + 8x_4$

We want to minimize C, and of course, this transportation problem can be solved by using Simplex Method!

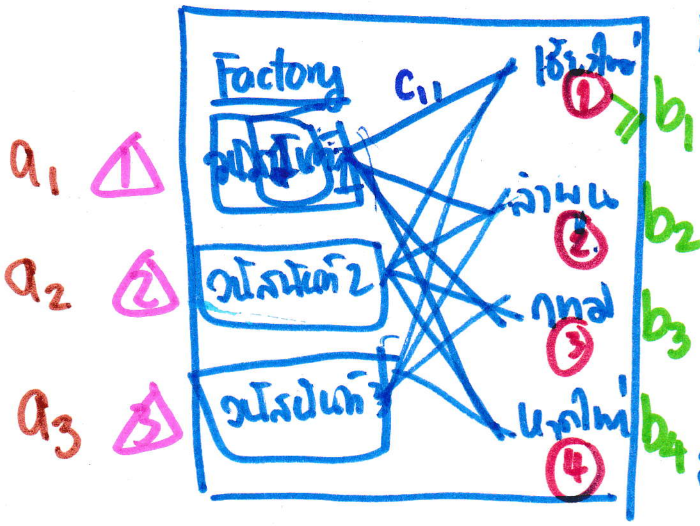
However, there is a special-purpose algorithm which is called **the transportation algorithm.** The algorithm consists of three stages using several alternative methods.

For example, Northwest Corner Rule, Minimum Cell-Cost Method and MODI (modified distribution) Method.

The transportation problem is only a special type of the LP problems.

There are computer programs which help in the construction of the LP or TP model. The best known is GAMS—General Algebraic Modeling System. The system is useful with large, complex problems. GAMS is available for use on personal computers, workstations, mainframes and supercomputers.

For example see
http://www.gams.com/docs/example.htm and
http://www.gams.com/
Note that we will only use Simplex Method in this course!



x_{ij} : # ของสินค้าที่ส่งจาก โรงงานผลิตภัณฑ์ i ไปยังจุดขาย j

X_{12} = ____________

X_{34} = ____________

สมมติว่า c_{ij} เป็นต้นทุนต่อหน่วยของสินค้า ในการส่งจากโรงงาน i ไปยังจุดขาย j

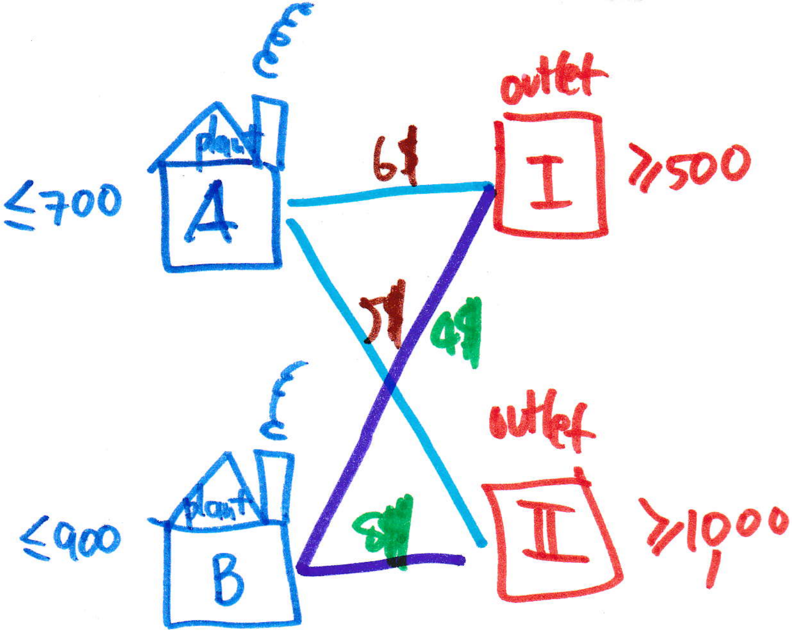
$$\text{Min } C = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} \rightarrow \text{โรงงานที่ 1}$$
$$+ c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} \rightarrow \text{โรงงานที่ 2}$$
$$+ c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34} \rightarrow \text{โรงงานที่ 3}$$

s.t.

$$\left.\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &\le a_1 \\ x_{21} + x_{22} + x_{23} + x_{24} &\le a_2 \\ x_{31} + x_{32} + x_{33} + x_{34} &\le a_3 \end{aligned}\right\} \text{ศักยภาพโรงงาน}$$

$$\left.\begin{aligned} x_{11} + x_{21} + x_{31} &\ge b_1 \\ x_{12} + x_{22} + x_{32} &\ge b_2 \\ x_{13} + x_{23} + x_{33} &\ge b_3 \\ x_{14} + x_{24} + x_{34} &\ge b_4 \end{aligned}\right\} \text{จ.น.สินค้าที่ stock ได้}$$

$x_{ij} \ge 0$ สำหรับ $i=1,\ldots 3$, $j=1,\ldots 4$



สมมติว่า X_{A1} เป็นจ.น. Comp ที่ส่งจาก plant A ไปยัง outlet I

X_{A2} 〃 ———————— 〃 A ไปยัง outlet II

X_{B1} 〃 ———————— 〃 B ไปยัง outlet I

X_{B2} 〃 ———————— B ไปยัง outlet II

$$\text{Min } C = 6X_{A1} + 5X_{A2} + 4X_{B1} + 8X_{B2}$$

s.t.

$$\left.\begin{aligned} X_{A1} + X_{A2} &\leq 700 \\ X_{B1} + X_{B2} &\leq 900 \end{aligned}\right\} \text{เงื่อนไขของ plant}$$

$$\left.\begin{aligned} X_{A1} + X_{B1} &\geq 500 \\ X_{A2} + X_{B2} &\geq 1{,}000 \end{aligned}\right\} \text{เงื่อนไข outlet}$$

โดยที่ $X_{ij} \geq 0$ สำหรับ $i = 1, 2$, $j = 1, 2$