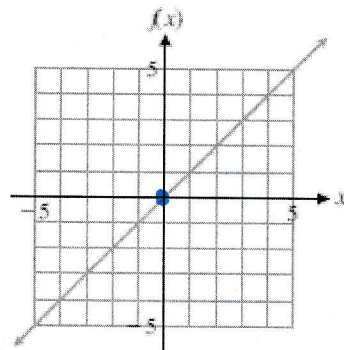
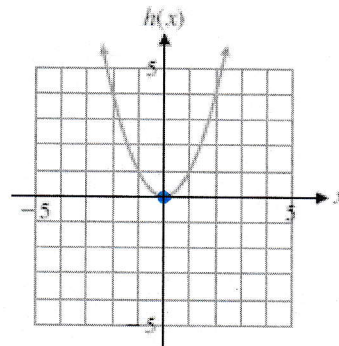


Example of Frequently Used Functions

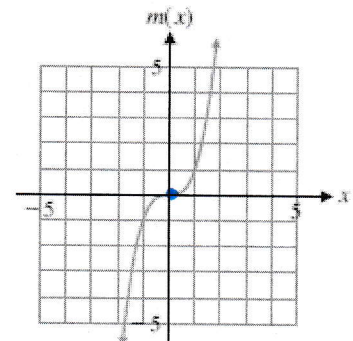
This figure shows the graph, range and domain of each of the basic elementary functions.



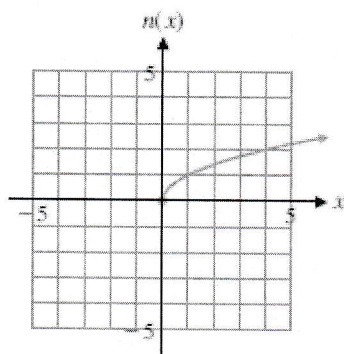
(A) Identity function
 $f(x) = x$
Domain: $\mathbb{R}$
Range: $\mathbb{R}$



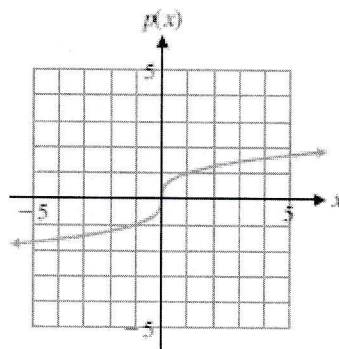
(B) Square function
 $h(x) = x^2$
Domain: $\mathbb{R}$
Range: $[0, \infty)$



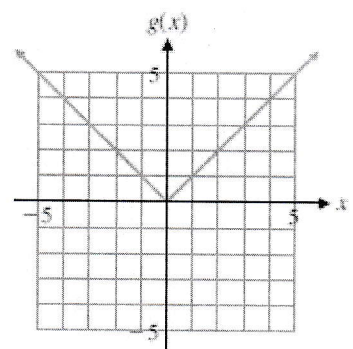
(C) Cube function
 $m(x) = x^3$
Domain: $\mathbb{R}$
Range: $\mathbb{R}$



(D) Square root function
 $n(x) = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$



(E) Cube root function
 $p(x) = \sqrt[3]{x}$
Domain: $\mathbb{R}$
Range: $\mathbb{R}$



(F) Absolute value function
 $g(x) = |x|$
Domain: $\mathbb{R}$
Range: $[0, \infty)$

Figure 1 Some basic functions and their graphs*

$$\sqrt[3]{-1} = -1$$

$$\sqrt[3]{-8} = -2$$

$$\textcircled{F} \quad | -3 | = 3$$

$$| 3 | = 3$$

Vertical Translation: $y = f(x) + k$

- $k > 0$ Shift graph of $y = f(x)$ up k units.
- $k < 0$ Shift graph of $y = f(x)$ down $|k|$ units.

Horizontal Translation: $y = f(x + h)$

- $h > 0$ Shift graph of $y = f(x)$ left h units.
- $h < 0$ Shift graph of $y = f(x)$ right $|h|$ units.

Note: Horizontal translation must be performed **before** any other transformations.

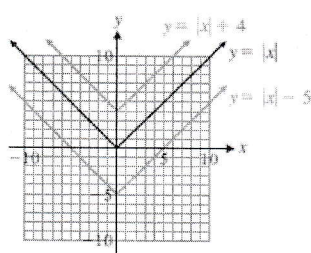


Figure 2 Vertical shifts

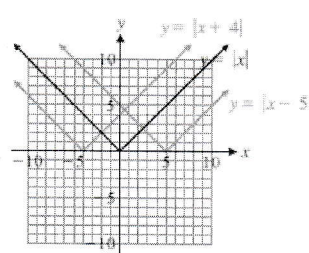


Figure 3 Horizontal shifts

Vertical Reflection: $y = -f(x)$

Reflect the graph of $y = f(x)$ in the x axis.

Vertical Stretch: $y = Af(x)$

Stretch graph of $y = f(x)$ vertically by multiplying each y -coordinate value by A .

When $0 < A < 1$, the graph is shrunk. When $A > 1$, the graph is stretched.

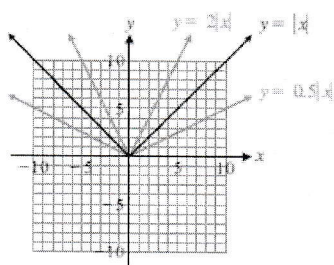


Figure 6 Vertical stretch and shrink

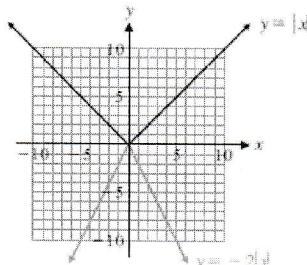
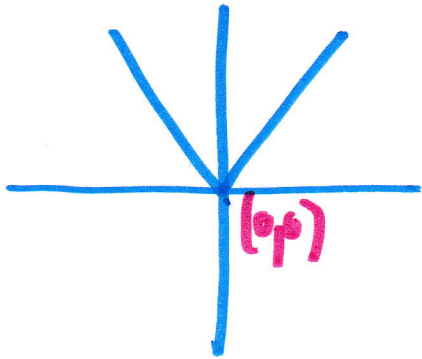


Figure 7 Reflection and vertical stretch

$$y = f(x) \quad - \quad \text{จุดศูนย์กลาง } (0,0)$$

$$y - k = f(x - h) \quad \text{เลื่อนที่จุด } (h, k)$$

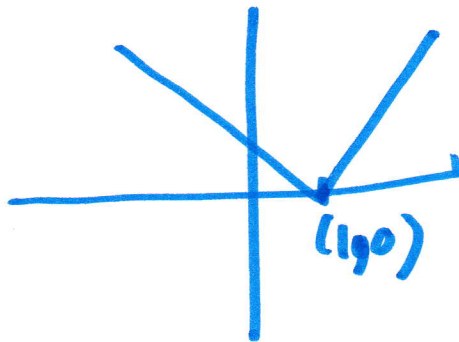
Ex) $y = |x|$



$$y = |x - 1|$$

$$y - 0 = f(x - 1)$$

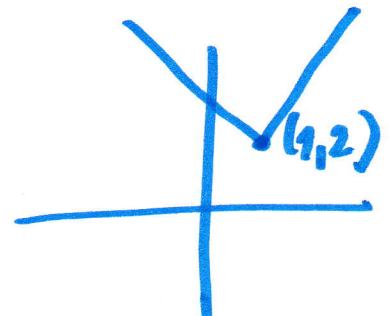
$$(0,0) \rightarrow (1,0)$$



$$y = |x - 1| + 2$$

$$y - 2 = f(x - 1)$$

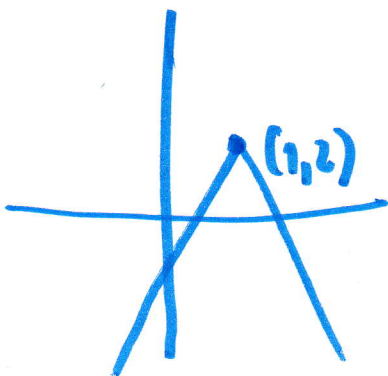
$$(0,0) \rightarrow (1,2)$$



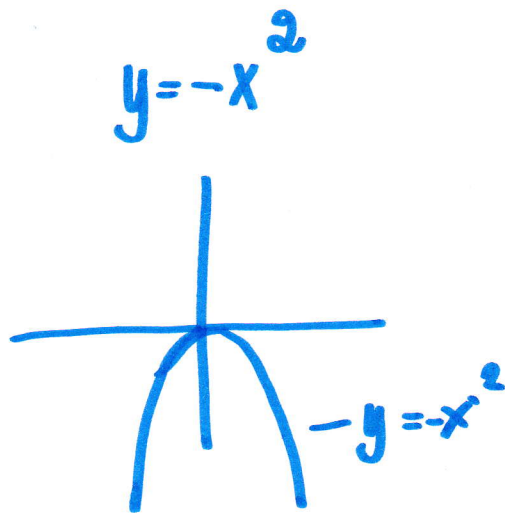
$$y = -|x - 1| + 2$$

$$y - 2 = -|x - 1| \quad \bullet (0,0) \rightarrow (1,2)$$

• กลับหน้า



$$y = -f(x)$$



$$y = f(x)$$

$y = Af(x)$, A ជា ឥរិយាបថ ១៧

 $A > 1$ 

ภาพ ๗๖:

นมสาว จ๊อบจ๋า
นิพนธ์, พงษ์ภักดิ์

$$0 < A < 1$$

→

9. ปรากฏการณ์

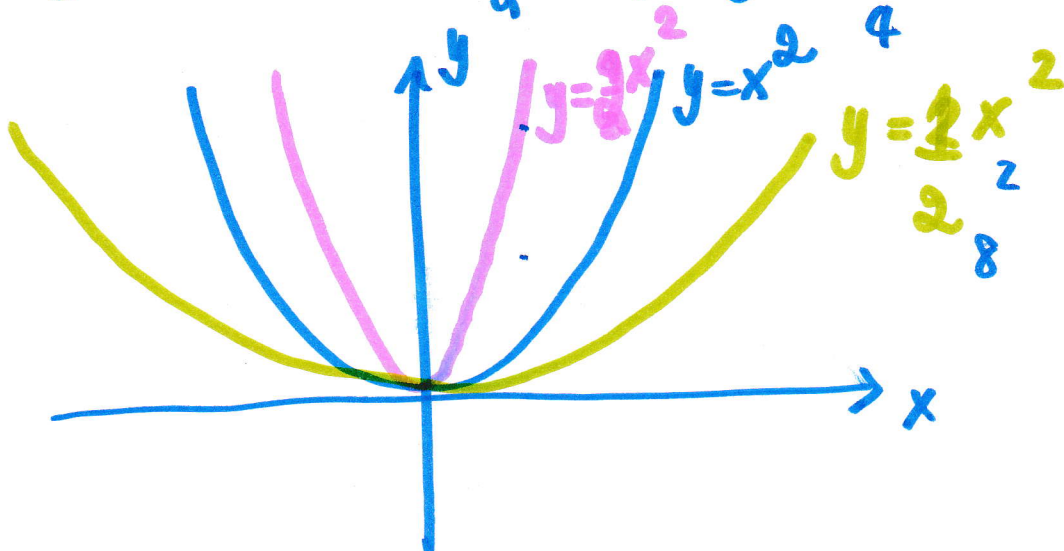
• உயரம், நிறம்

Ex

$$y = x^2$$

$$y = \frac{1}{2}x^2$$

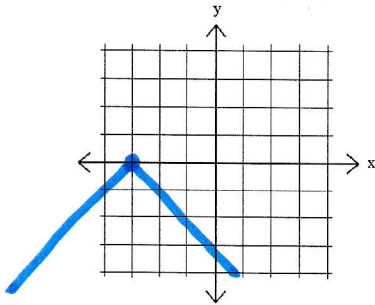
$$y = 2x^2$$



Example 5 Sketch the graph of each function and find the domain and range.

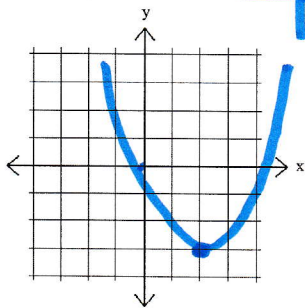
1. $f(x) = -|x + 3|$ $y = |x|$ ✗ $y - 0 = -f(x+3)$

- จุด $(0,0)$ เลื่อนไปที่ $(-3,0)$
- ค่าแนวทแยง $(-)$ กลับหัว.



2. $g(x) = (x - 2)^2 - 3$ $y = x^2$ ✗ $y + 3 = (x-2)^2$

- จุด $(0,0)$ เลื่อนไปที่ $(2,-3)$

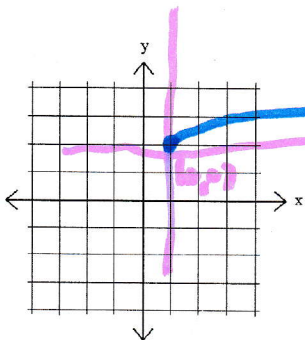


$y = h(x)$

3. $h(x) = 2\sqrt{x-1} + 2$ $y = \sqrt{x}$ ✗ $y - 2 = 2\sqrt{x-1}$

$y - 2 = 2f(x-1), f(x) = \sqrt{x}$

- จุด $(0,0)$ เลื่อนไปที่ $(1,2)$
- $A = 2 > 1$ ขยาย



Example 6 Match each function with its graph.

1. $f(x) = -4x^2 + 5$ **g**

4. $f(x) = -\sqrt{x}$ **h**

7. $f(x) = |x| + 1$ **d**

2. $f(x) = x + 1$ **c**

5. $f(x) = \frac{1}{2}x^3 + 1$ **b**

8. $f(x) = |x + 1|$

3. $f(x) = x^3 - 1$ **f**

6. $f(x) = \sqrt{x} + 1$ **a**

e ⑦ $y - 1 = |x|$

$(0,0) \rightarrow (0,1)$

~~$y = |x|$~~

⑧ $f(x) = |x+1|$

$y - 0 = g(x+1)$

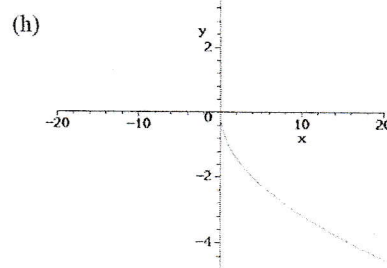
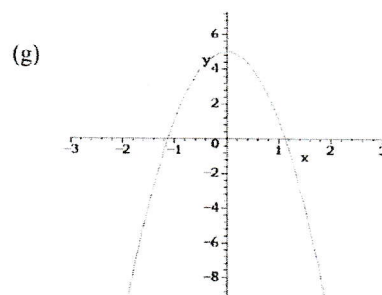
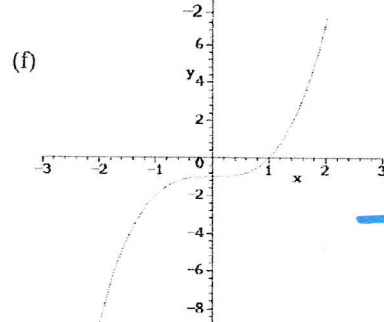
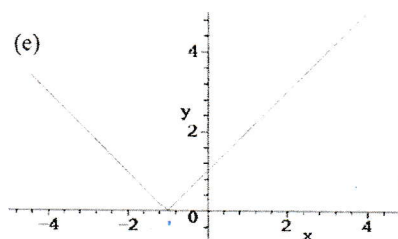
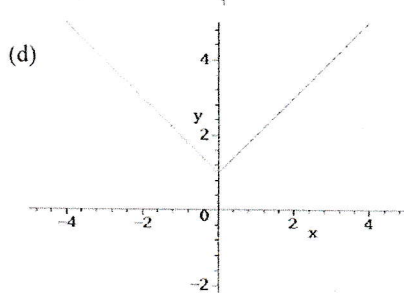
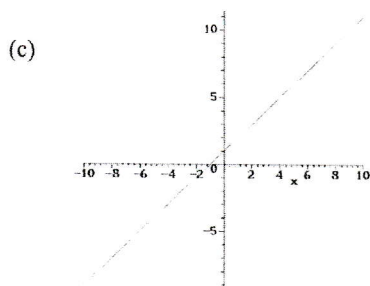
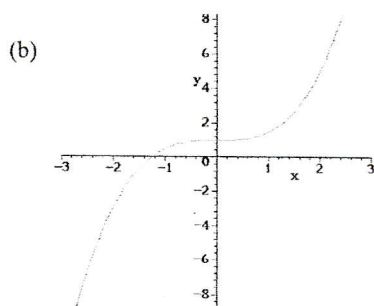
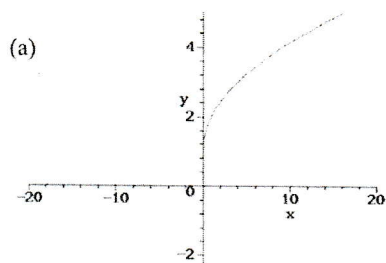
~~$g(x) = |x|$~~

$(0,0) \rightarrow (-1,0)$

$g(x) = |x|$

$g(1) = |1|$

$g(x+1) = |x+1|$

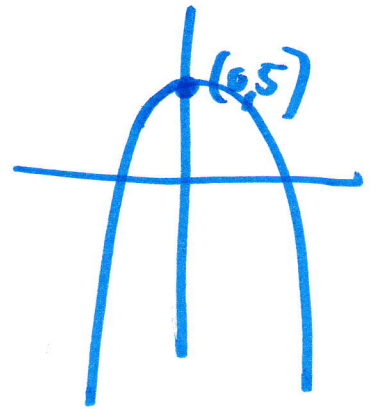


$$\textcircled{1} \quad f(x) = -4x^2 + 5 \quad \left| \quad \begin{array}{l} y = x^2 \\ y - 5 = -4x^2 \end{array} \right.$$

$$\textcircled{*} (0,0) \rightarrow (0,5)$$

$\textcircled{*}$ เครื่องหมาย (-) กลับหัวลง

$\textcircled{*}$ 4 จุดเข้า $\rightarrow$ กราฟปิดแล้ว

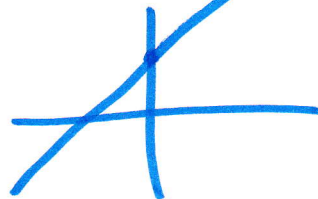


$$\textcircled{2} \quad f(x) = x + 1$$

$$y - 1 = x \rightarrow$$

$$\bullet (0,0) \rightarrow (0,1)$$

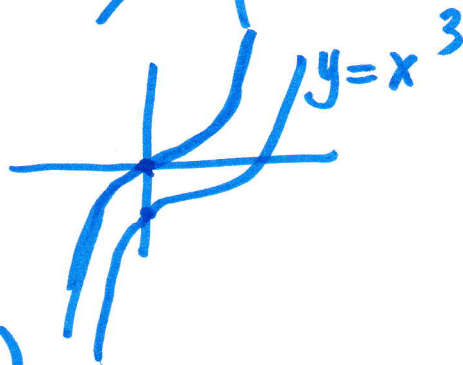
$$y = x$$



$$\textcircled{3} \quad f(x) = x^3 - 1$$

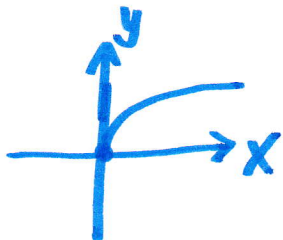
$$y + 1 = x^3$$

$$(0,0) \rightarrow (0,-1)$$

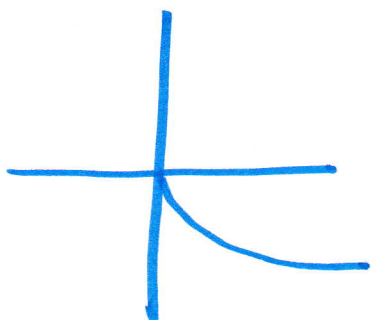


4) $f(x) = -\sqrt{x}$

$f(x) = \sqrt{x}$

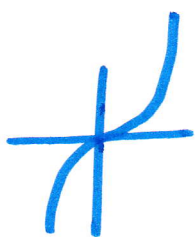


$f(x) = -\sqrt{x}$



5) $f(x) = \frac{1}{2}x^3 + 1$

$(g(x) = x^3)$



$y - 1 = \frac{1}{2}x^3$

$\Rightarrow (0,0) \rightarrow (0,1)$

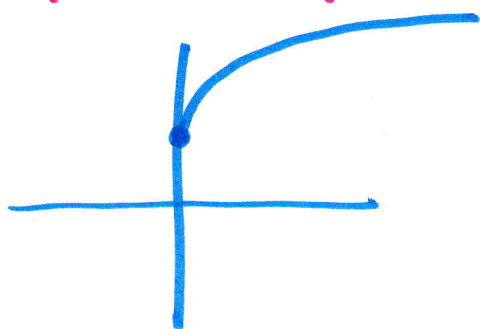
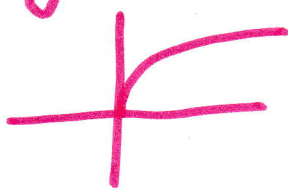
$\frac{1}{2}$ จุดต่อ $\Rightarrow$ กราฟย้ายขึ้น

6) $f(x) = \sqrt{x} + 1$

$y - 1 = \sqrt{x}$

$(0,0) \rightarrow (0,1)$

$y = \sqrt{x}$



Consider the absolute function $f(x) = |x|$. We can write it as the form

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

The absolute function can be considered as the combination of two parts, i.e. the part that $x \geq 0$ and $x < 0$. This kind of the combination is said to be the piecewise-defined function.

Piecewise-defined Function

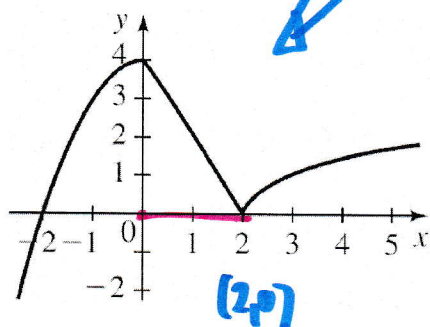
The piecewise-defined function is a function that the formula for a function f changes, depending on the value of x .

Example 7 Match each function with its graph.

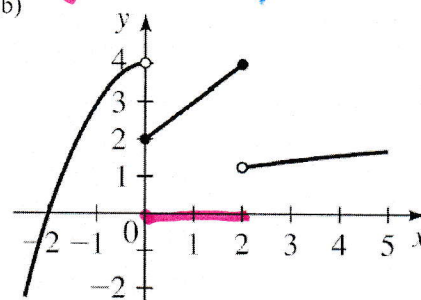
1. $f(x) = \begin{cases} 4 - x^2, & x < 0 \\ x + 2, & 0 \leq x \leq 2 \\ \sqrt[3]{x}, & x > 2 \end{cases}$

2. $f(x) = \begin{cases} 4 - x^2, & x < 0 \\ 4 - 2x, & 0 \leq x \leq 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$

(a)

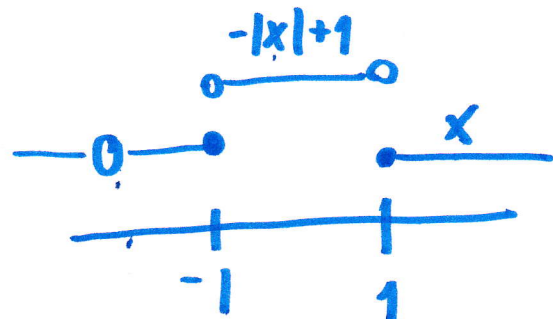


(b)

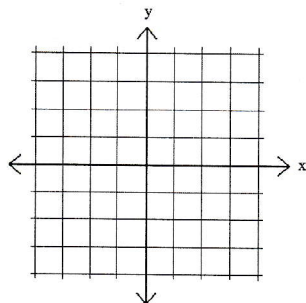


Example 8 Define the piecewise-defined function f by

$$f(x) = \begin{cases} 0, & x \leq -1 \\ -|x| + 1, & -1 < x < 1 \\ x, & x \geq 1 \end{cases}$$



Sketch the graph of $f(x)$, and compute $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, $f(2)$.



$$\begin{aligned} f(-2) &= 0 & f(2) &= 2 \\ f(-1) &= 0 \\ f(0) &= 1 \\ f(1) &= 1 \end{aligned}$$

Quadratic Functions:

A quadratic function is a function that can be written in the standard form $f(x) = ax^2 + bx + c$ where a, b, c are real numbers with $a \neq 0$.

Note that the quadratic function can be transformed into the vertex form

$$f(x) = a(x - h)^2 + k.$$

Then the graph of f is a parabola with the vertex (h, k) , which can be represented as the coefficient of the standard form by

$$(h, k) = \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right).$$

We can derive it using the completing of the square.

- If $a > 0$, the parabola opens upward and if $a < 0$, the parabola opens downward.
- The domain of f is the set of all real numbers.
- If $a > 0$, the range of f is $\{y \mid y \geq k\}$ and if $a < 0$, the range of f is $\{y \mid y \leq k\}$.
- The axis of symmetry is $x = h$.

Note: Similar analysis can be applied on the vertex forms of square root functions and absolute functions.

