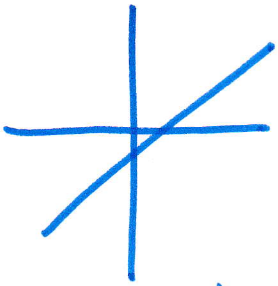
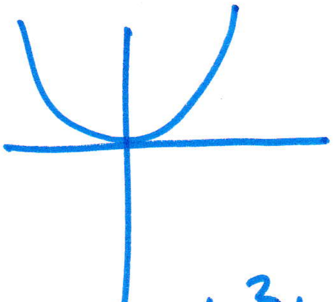
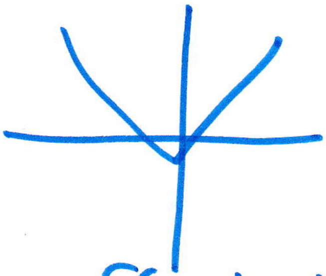


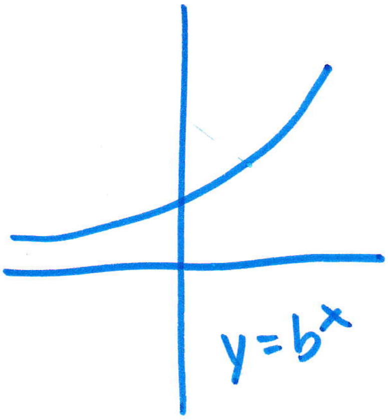
$y=mx+b$



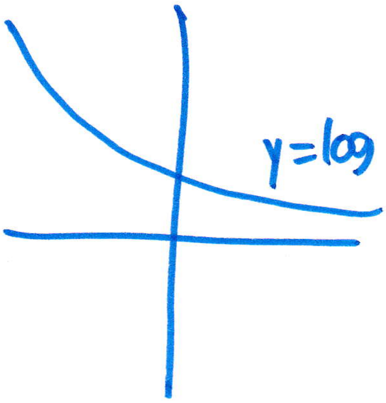
$y=a(x-h)^2+k$



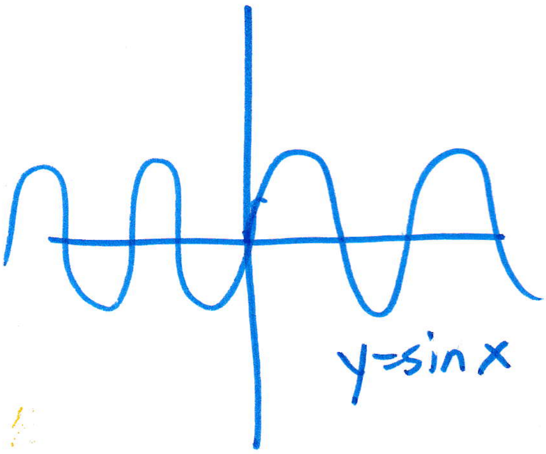
$f(x)=|x-h|+k$



$y=b^x$



$y=\log_b x$



$y=\sin x$

← Trigonometric function

Trigonometric Functions

Angles: Radian Measure: A unit circle is a circle with radius of length one. On a unit circle, a central angle subtended by an arc of length θ is said to have **radian measure** θ, written as θ **radian** or θ **rad**.

A central angle of 180° is subtended by an arc that is 1/2 the circumference of a unit circle. Thus, it has radian measure $\frac{1}{2}(2\pi 1) = \pi$.

In general, the formula

$$\frac{\theta_{\text{deg}}}{180} = \frac{\theta_{\text{rad}}}{\pi}$$

can be used to convert degree measure to radian measure and vice versa.

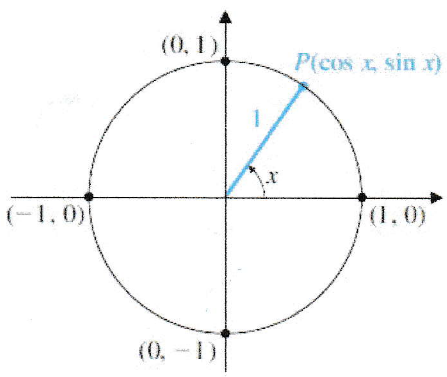
Example 12 Find the radian measure of

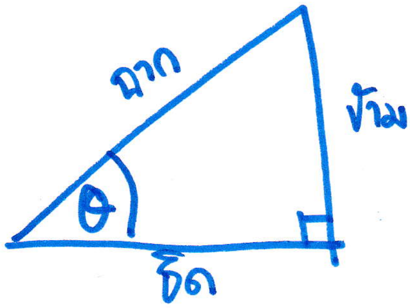
1. $0\,^\circ$
2. $30\,^\circ$
3. $45\,^\circ$
4. $60\,^\circ$
5. $90\,^\circ$
6. $360\,^\circ$

Sine and Cosine Functions: Consider a unit circle in a coordinate system with center at the origin and a central angle of size θ rad between the positive side of the x-axis and the other radius, called R. Let P be the end of the radius R on the circle.

- The x-coordinate of P is called the **cosine of** θ (abbreviated $\cos\theta$).
- The y-coordinate of P is called the **sine of** θ (abbreviated $\sin\theta$).

The set of all ordered pairs of the form $(\theta, \cos\theta)$ and the set of all ordered pairs of the form $(\theta, \sin\theta)$ constitute, respectively, the **cosine function** and **sine function**.

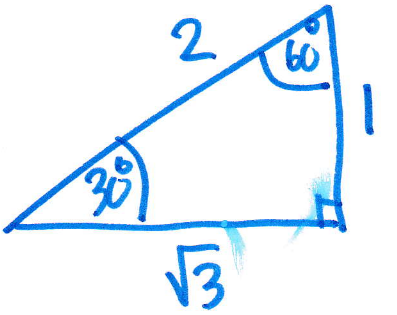


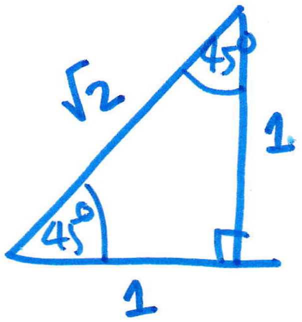


$\sin\theta = \dfrac{\text{ข้าม}}{\text{ฉาก}}$ $\qquad \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

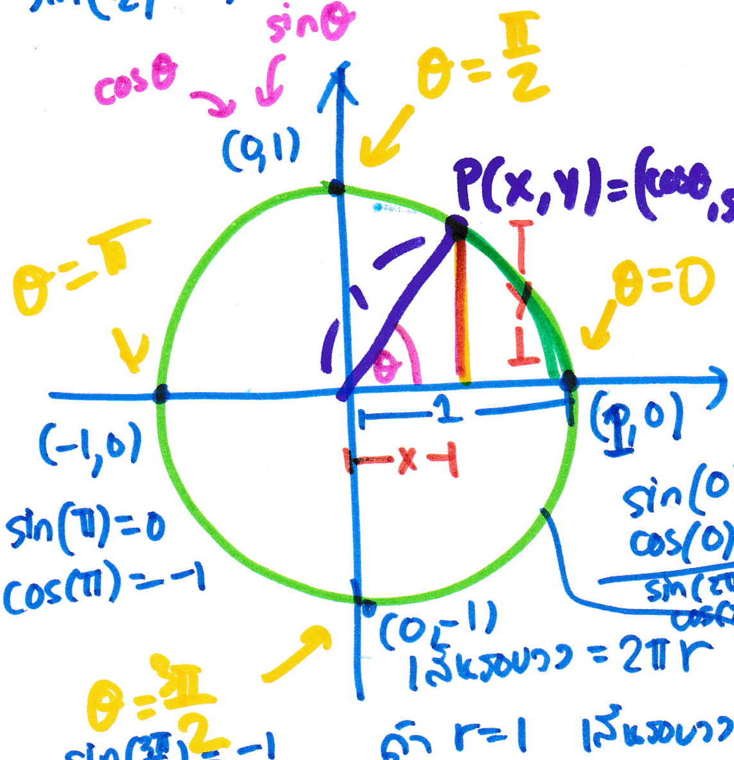
$\cos\theta = \dfrac{\text{ชิด}}{\text{ฉาก}}$ $\qquad \sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\text{ข้าม}}{\text{ชิด}} = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$





$\sin(\frac{\pi}{2}) = 1, \cos(\frac{\pi}{2}) = 0$



มุมในหน่วยเรเดียน

$\sin\theta = \dfrac{y}{1}$, $\cos\theta = \dfrac{x}{1}$ $\Rightarrow (x,y) = (\cos\theta, \sin\theta)$

$\sin(\pi) = 0$

$\cos(\pi) = -1$

$\sin(0) = 0$

$\cos(0) = 1$

$\sin(2\pi) = 0$

$\cos(2\pi) = 1$

$\sin(\frac{3\pi}{2}) = -1$

$\cos(\frac{3\pi}{2}) = 0$

เส้นรอบวง $= 2\pi r$

ถ้า $r = 1$ เส้นรอบวง $= 2\pi$

มุมรอบจุด 360° ความยาวส่วนโค้ง 2π

มุมรอบจุด x° ความยาวส่วนโค้ง $\frac{2\pi}{360} \times x = \frac{\pi}{180} \times x$ เรเดียน

มีมุม $45^\circ = \frac{\pi}{180} \cdot 45 = \frac{\pi}{4}$ เรเดียน

Ex มี $\frac{\pi}{6}$ เรเดียน เป็นกี่องศา

2π เรเดียนเป็น 360°

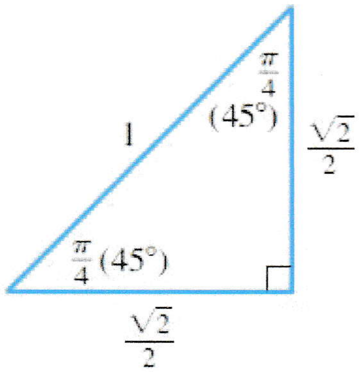
$\frac{\pi}{6}$ เรเดียนเป็น $\frac{360^\circ}{(2\cancel{\pi})}\left(\frac{\cancel{\pi}}{6}\right) = 30^\circ$

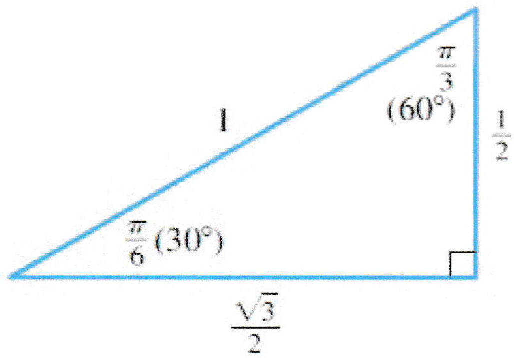
The domain of these two functions is the set of all angles, with measure either in degrees or radians. The range is $[-1, 1]$.

Example 13 Evaluating the following sine and cosine functions.

1. $\cos 90°$
2. $\sin 180°$
3. $\cos \pi$
4. $\cos(2\pi)$

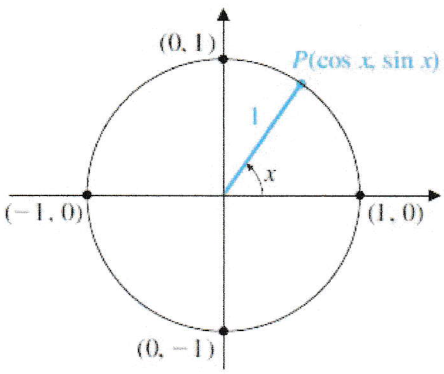
Exact values of the sine and cosine functions can be obtained for multiples of the special angles, because these triangles can be used to find the coordinate of the intersection of the terminal side of each angle with the unit circle.

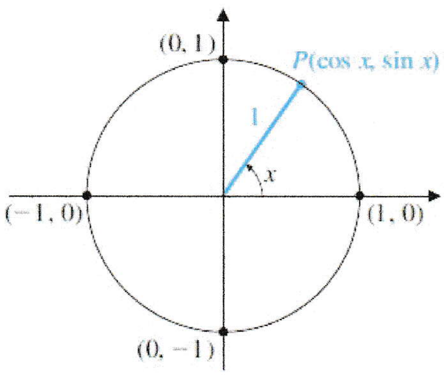




Example 14 Find the exact value of each of the following:

1. $\cos(\frac{\pi}{4})$
2. $\sin(\frac{\pi}{6})$
3. $\sin(\frac{\pi}{4})$
4. $\cos(\frac{\pi}{3})$





Four Other Trigonometric Functions: The other four trigonometric functions are the **tangent, cotangent, secant** and **cosecant**.

- $\tan x = \frac{\sin x}{\cos x}, \cos x \neq 0$
- $\cot x = \frac{\cos x}{\sin x}, \sin x \neq 0$
- $\sec x = \frac{1}{\cos x}, \cos x \neq 0$
- $\operatorname{cosec} x = \frac{1}{\sin x}, \sin x \neq 0$

The sine and cosine functions are only two of six trigonometric functions. They are, however, the most important of the six for many applications.

Example 15 Find the exact value of each in this table:

x	$\sin x$	$\cos x$	$\tan x$	$\cot x$	$\sec x$	$\operatorname{cosec} x$
0						
$\frac{\pi}{6}$						
$\frac{\pi}{4}$						
$\frac{\pi}{2}$						

Exponential Functions: An exponential function is a function that can be written in the form

$$f(x) = b^x, \quad \text{where} \quad b > 0 \text{ and } b \neq 1$$

The domain of f is the set of all real numbers and the range of f is the set of all positive real numbers.
The exponential function with base e, $f(x) = e^x$, is used more frequently than all other bases combined, where e is an irrational number that can be approximated as 2.71828.

Function

ฟังก์ชันพหุนาม : $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Ex

$P(x) = 5x^2 + 4x + 6$ $\quad$ $\deg P(x) = 2$

$P(x) = 5$ ← $5 \cdot x^0$ $\quad$ $\deg P(x) = 0$

$Q(x) = 5x^{-1}$ ไม่เป็นฟังก์ชันพหุนาม.

ฟังก์ชัน Exponential $\boxed{f(x) = b^{x}}$, x ตัวแปร, b เป็นตัวเลข

Ex

- $f(x) = 5^x$

$f(1) = 5^1$ $\quad|\quad$ $f(-2) = 5^{-2} = \frac{1}{5^2}$

$f(100) = 5^{100}$ $\quad|\quad$ $f(\frac{1}{2}) = 5^{\frac{1}{2}} = \sqrt{5}$

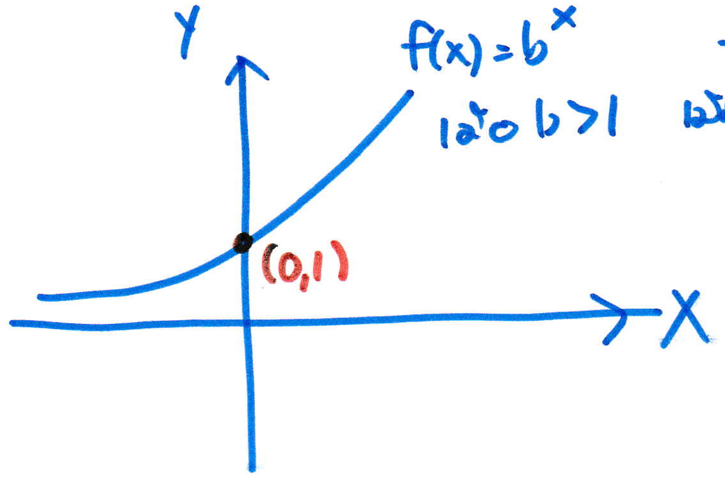
- $f(x) = (\frac{1}{5})^x$

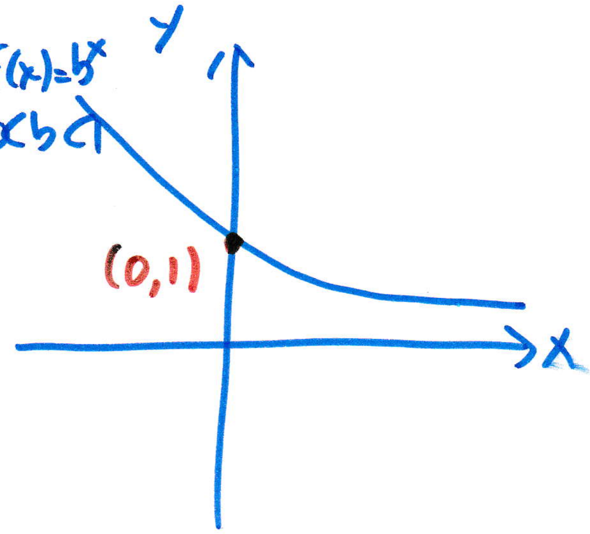
$f(1) = (\frac{1}{5})^1 = \frac{1}{5}$ $\quad|\quad$ $f(-2) = (\frac{1}{5})^{-2}$

$f(100) = (\frac{1}{5})^{100} =$ $\quad|\quad$ $f(0) = (\frac{1}{5})^0 = 1$

$f(x) = e^x$

↑ จำนวนอตรรกยะ: ≈ 2.71828





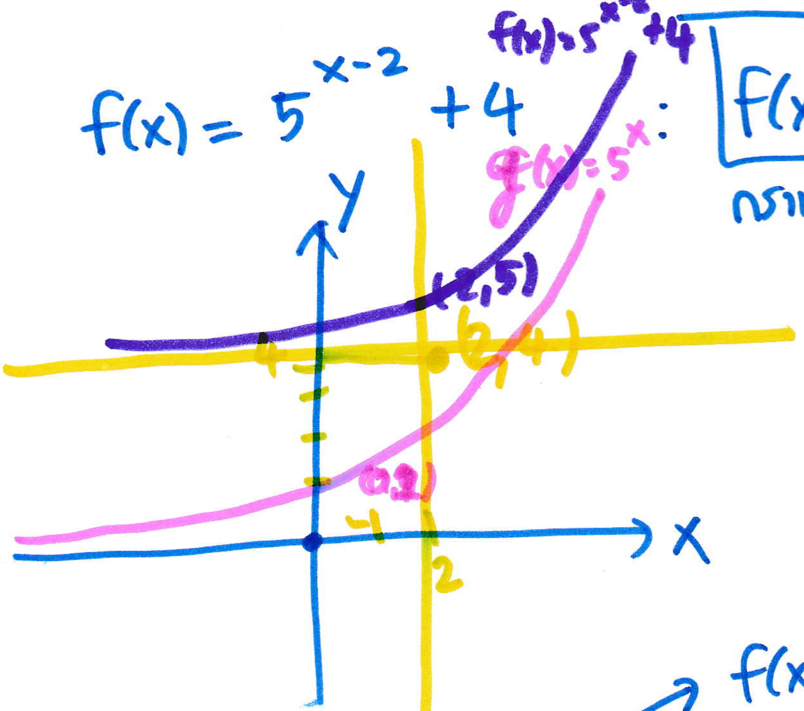
จุดตัดแกน y ให้ $x = 0$

$$f(0) = b^0 = 1$$

$$f(x) = 5^{x-2} + 4 :$$

$$\boxed{f(x) - k = 5^{(x-h)}}$$

กราฟจะถูกย้ายจุดตัวอ้างอิงทั้งหมดไปยัง
จุด $(2, 4)$

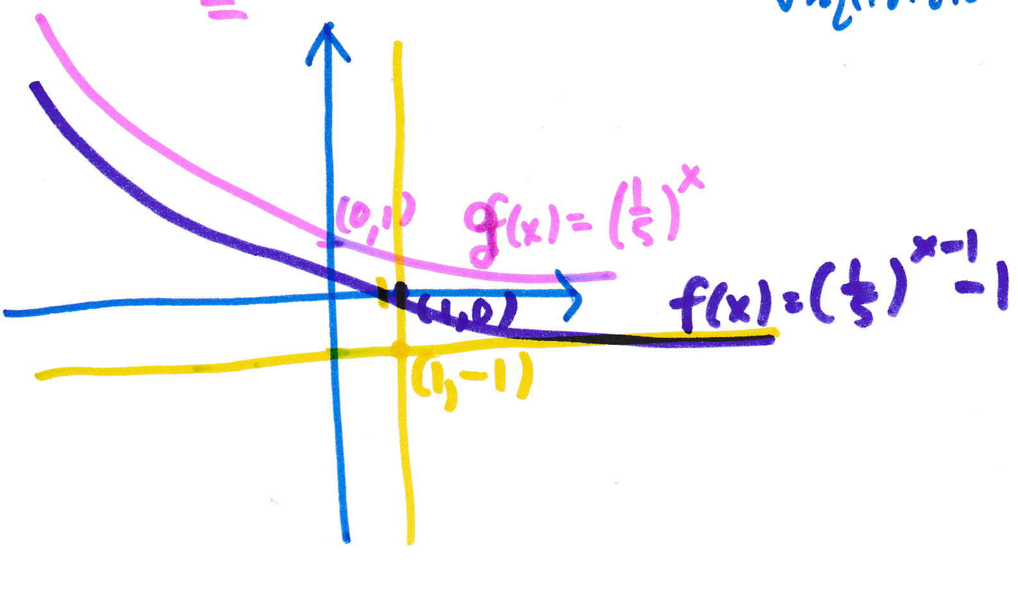


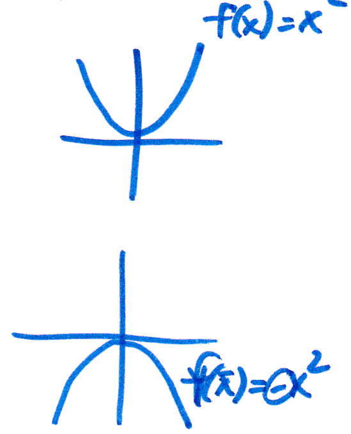
$$f(x) - k = \left(\frac{1}{5}\right)^{x-1}$$

$$f(x) + 1 = \left(\frac{1}{5}\right)^{x-1}$$

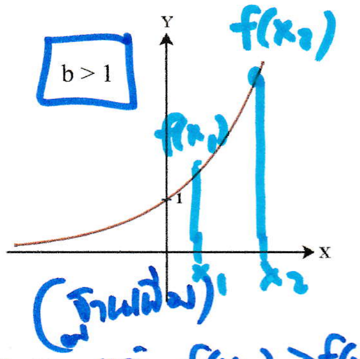
$$f(x) = \left(\frac{1}{5}\right)^{x-1} - 1$$

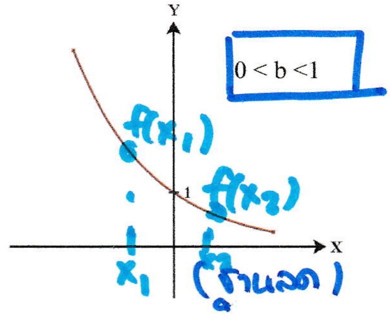
→ จาก elementary function
จะถูกย้ายไป $(1, -1)$





The Graphs of Exponential Functions $f(x)=b^x$





ถ้า $x_2 \geq x_1$ แล้ว $f(x_2) \geq f(x_1)$

ถ้า $x_2 \geq x_1$ แล้ว $f(x_2) \leq f(x_1)$

> **Properties of Exponential Functions**
>
> For a and b positive, $a \neq 1$, $b \neq 1$, and x and y real,
>
> 1. $a^x a^y = a^{x+y}$ and $\dfrac{a^x}{a^y} = a^{x-y}$,
> 2. $(a^x)^y = a^{xy}$ and $(ab)^x = a^x b^x$,
> 3. $a^x = a^y$ if and only if $x = y$,
> 4. For $x \neq 0$, $a^x = b^x$ if and only if $a = b$.
>
> **Note:** Properties 1 and 2 come directly from the definition of exponent.

$\boxed{(a+b)^x \neq a^x + b^x}$

Example 16 Solve each equation for x.

1. $\dfrac{5^{2x}5^7}{5^3} = 5^{4x}$

$\dfrac{5^{2x+7}}{5^3} = 5^{4x}$ (PP.1)

$5^{(2x+7)-3} = 5^{4x}$ (P.P.1)

$5^{2x+4} = 5^{4x}$ (PP.3)

$2x+4 = 4x \Rightarrow \boxed{x=2}$

2. $5^3 = (x+2)^3$

จาก(PP4) $5 = x+2$

$\boxed{x=3}$

3. $(10^x)^2 = 2^6 5^6$

$(10^x)^2 = (10)^6$ PP.2

$10^{2x} = 10^6$ PP. 2

$2x = 6$ PP. 3

$\boxed{x=3}$

$\sqrt{10} \cdot (10^x)^2 = 2^6 5^6$

$10^{\frac{1}{2}}$

$10^{\frac{1}{2}} \cdot 10^{2x} = (10)^6$

$2^{\sqrt{2}}$

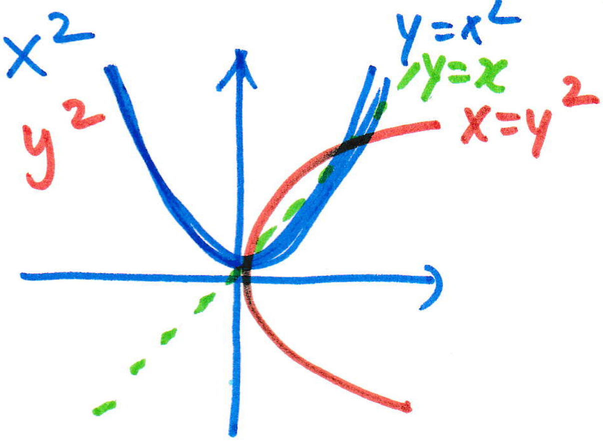
$$\boxed{r = \{(x,y) \in \mathbb{R}\times\mathbb{R} \mid y = 2x\}}$$

$r_1 = \{(1,2), (2,4), (3,6)\}$

$r_1^{-1} = \{(2,1), (4,2), (6,3)\}$

$f(x) = x^2 \longrightarrow y = x^2$

$x = y^2$



$y = b^x$: สลับ $x, y \Rightarrow x = b^y$

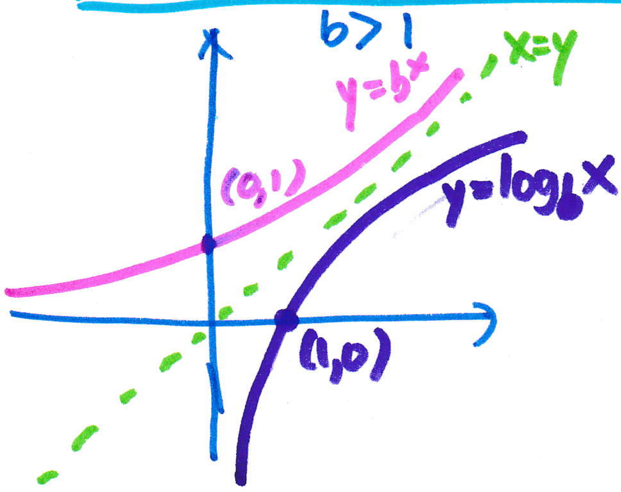
จริงแล้วหาได้ $\boxed{y = ?}$

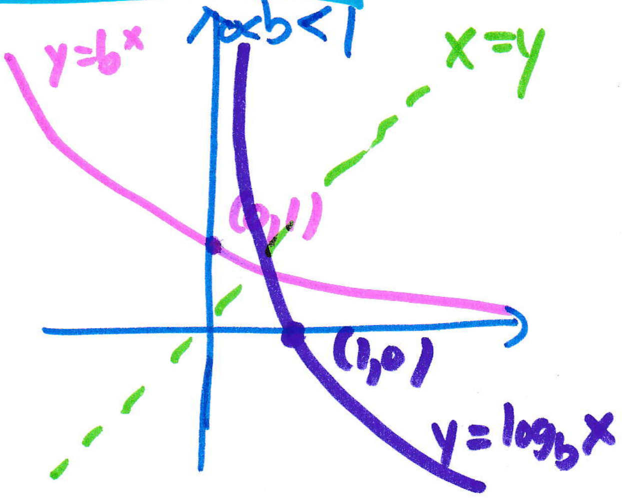
$$\boxed{y = \log_b x}$$

logarithmic function

$$\boxed{y = \log_b x \iff x = b^y}$$

$$\boxed{\log_{\text{ฐาน}} x = \text{ริกเตอ} \iff x = \text{ฐาน}^{\text{ริกเตอ}}}$$





Logarithmic Functions: A logarithmic function is a function that is closely related to an exponential function. For $b > 0$ and $b \neq 1$,

$$\boxed{y = \log_b x \quad \text{is equivalent to} \quad x = b^y}$$

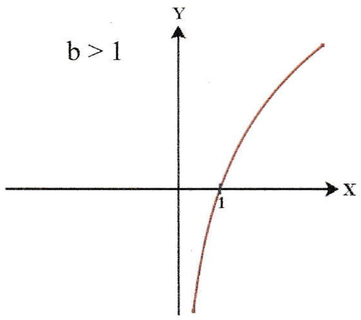
The domain of f is the set of all positive real numbers and the range of f is the set of all real numbers.

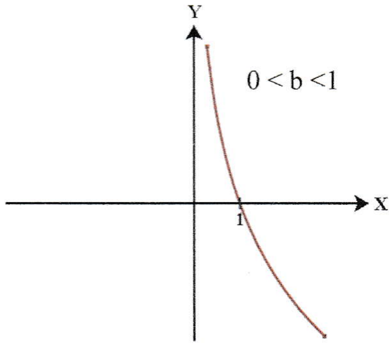
If $b = 10$, we can omit writing the base, i. e., $\log x = \log_{10} x$.

If $b = e$, we use notation $\ln x$ for the logarithm with base e, i.e., $\ln x = \log_e x$.

Any logarithms with base e are called the **natural logarithms**.

The Graphs of Logarithmic Functions





Properties of Logarithmic Functions

If b, M and N are positive real numbers, $b \neq 1$, and p and x are real numbers, then

1. $\log_b 1 = 0$ $\equiv$ $b^0 = 1$
2. $\log_b b = 1$ $\equiv$ $b^1 = b$
3. $\log_b b^x = x$ $\equiv$ $b^x = b^x$
4. $b^{\log_b x} = x$ $\equiv$ $\log_b x = \log_b x$
5. $\log_b MN = \log_b M + \log_b N$ $\equiv$ $b^x \cdot b^y = b^{x+y}$
6. $\log_b \dfrac{M}{N} = \log_b M - \log_b N$ $\equiv$ $\dfrac{b^x}{b^y} = b^{x-y}$
7. $\log_b M^p = p \log_b M$
8. $\log_b M = \log_b N$ if and only if $M = N$

$\boxed{\log_b b^x = x \log_b b = x}$

Note: Properties 1-7 come directly from the definition of exponent.

Ex $5^{\log_5 10} = 10$

ข้อควรระวัง

$\log_b (A \pm B) \neq \log_b A \pm \log_b B$

$(\log_b A)^k \neq k \log_b A$

$y = \log x \iff y = \log_b x$ เมื่อ $b = 10$

$\boxed{y = \ln x} \iff y = \log_e x$ เมื่อ $b = e$

natural logarithm

$$\boxed{y = \log_b x \iff x = b^y}$$

x-intercept : ให้ $y = 0$

$\therefore\ 0 = \log_b x \iff x = b^0 = 1$

$\therefore$ จุดตัดแกน x คือ $(1,0)$

$\log_{\text{ฐาน}} x = \text{ตัวชี้} \Leftrightarrow x = \text{ฐาน}^{\text{ตัวชี้}}$

Example 17 Change each exponential form to an equivalent logarithmic form.

1. $4^{\frac{1}{2}} = 2$ (b y x)

$\log_4 2 = \frac{1}{2}$

2. $r^s = t$

$s = \log_r t$

Example 18 Let $\log a = 2$ and $\log b = 3$, find

1. $\log 10a^3$ $= \log 10 + \log a^3$
$= 1 + 3\log a$
$= 1 + 3(2)$
$= 7$ #

2. $\log \frac{\sqrt{a}}{b}$ $= \log \sqrt{a} - \log b$
$= \log a^{\frac{1}{2}} - \log b$
$= \frac{1}{2}\log a - \log b$
$= \frac{1}{2}(2) - 3 = -2$ #

3. $\log(2^{\log_2 b})$ $= \log(b)$
$= 3$ #

Example 19 Solve the equation $\boxed{\ln(x+1) + \ln 3} = \ln 24$.

PP.5. $\Rightarrow \ln(\underline{3(x+1)}) = \ln(24)$

PP.8 $\Rightarrow 3(x+1) = 24$
$3x + 3 = 24$
$3x = 21$
$x = 7.$ #