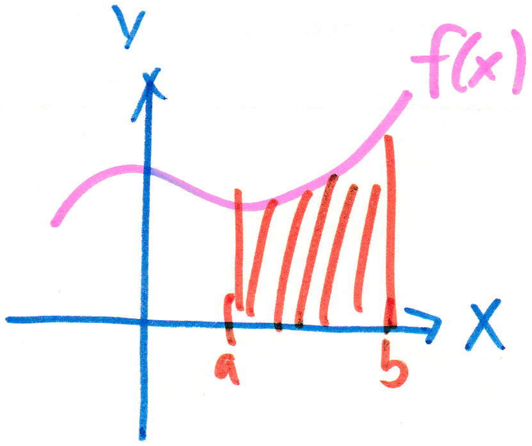
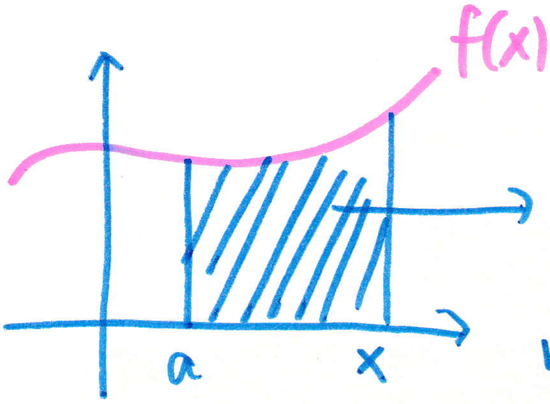


$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=0}^{n-1} f(x_k^*)\Delta x$$

$$= \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k$$



$$A(x) = \int_a^x f(x)\,dx$$

พยายามแสดงว่า $A(x)$ คืออะไรที่

$$A'(x) = f(x)$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int (x^2 + \underset{(x^{\frac{1}{2}})}{\sqrt{x}})\,dx = \frac{x^3}{3} + \frac{2x^{\frac{3}{2}}}{3} + C$$

$$\int (4x+1)^{100}\,dx$$

$u = 4x+1$

เป้าหมาย: $\int u^{100}\,du$

$du = 4dx \Rightarrow dx = \frac{1}{4}du$

$$\hookrightarrow \int u^{100}\left(\frac{1}{4}\right)du = \frac{1}{4}\int u^{100}\,du = \frac{1}{4}\cdot\frac{u^{101}}{101} + C$$

$$= \frac{1}{4}\left(\frac{(4x+1)^{101}}{101}\right) + C \quad \#$$

5.5 Integration by Substitution

Theorem 5.8 Let u be a function of x and f be a function of u. Then

$$\int [f(u)]du = \int [f(u(x))u'(x)]dx.$$

Example 5.10 Evaluate $\int \frac{x^2}{(3x^3-2)^9}dx$

เลือก $u = 3x^3 - 2 \Rightarrow du = 9x^2 dx \Rightarrow \boxed{dx = \frac{1}{9x^2}du}$

$$\int \frac{\cancel{x^2}}{u^9}\left(\frac{1}{9\cancel{x^2}}\right)du = \frac{1}{9}\int \frac{1}{u^9}du$$

$$= \frac{1}{9}\int u^{-9}du$$

$$= \frac{1}{9}\cdot\frac{u^{-9+1}}{(-9+1)} + C$$

$$= -\frac{1}{72}\cdot\frac{1}{u^8} + C$$

แทนค่าคืน
$$= -\frac{1}{72}\cdot\frac{1}{(3x^3-2)^8} + C$$

$$\therefore \int \frac{x^2}{(3x^3-2)^9}dx = -\frac{1}{72}\cdot\frac{1}{(3x^3-2)^8} + C \quad \#$$

Example 5.11 Evaluate $\int \cos(5x)dx$

เลือก $u = 5x \therefore du = 5dx \Rightarrow dx = \frac{du}{5}$

$$\int \cos(5x)dx = \int \cos u \frac{du}{5} = \frac{1}{5}\int \cos u \, du$$

$$= \frac{1}{5}\sin u + C$$

$$= \frac{1}{5}\sin(5x) + C \quad \#$$

Example 5.12 Evaluate $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}dx$

เลือก $u = \sqrt{x} \therefore du = \frac{1}{2\sqrt{x}}dx$

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}dx = 2\int \cos(\sqrt{x})\underbrace{\frac{dx}{2\sqrt{x}}}_{du}$$

$$= 2\int \cos u \, du$$

$$= 2\sin u + C$$

$$= 2\sin(\sqrt{x}) + C \quad \#$$

①

$$d(\sqrt{x}) = \frac{1}{2\sqrt{x}}dx$$

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}dx = 2\int \cos(\sqrt{x})\underbrace{\frac{dx}{2\sqrt{x}}}_{d(\sqrt{x})}$$

$$= 2\int \cos(\sqrt{x})\,d(\sqrt{x})$$

$$= 2\sin(\sqrt{x}) + C$$

$$\#$$

②

Example 5.13 Evaluate $\int e^x \sec^2(e^x+1)dx$

เลือก $u = e^x + 1 \quad \therefore du = e^x dx$

$$\therefore \int \underbrace{e^x \sec^2(e^x+1)dx}_{du} = \int \sec^2 u \, du$$

$$= \tan u + C$$

$$= \tan(e^x+1) + C \quad \#$$

Example 5.14 Evaluate $\int \frac{1}{\sqrt{2-x^2}}dx$

$$\int \frac{1}{\sqrt{2-x^2}}dx = \int \frac{1}{\sqrt{2(1-\frac{x^2}{2})}}dx = \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{1-\frac{x^2}{2}}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{1-(\frac{x}{\sqrt{2}})^2}}dx \quad du$$

เลือก $u = \frac{x}{\sqrt{2}} \quad \therefore du = \frac{1}{\sqrt{2}}dx$

$$\therefore \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{1-(\frac{x}{\sqrt{2}})^2}}dx = \int \frac{1}{\sqrt{1-u^2}}du$$

$$= \arcsin u + C$$

$$= \arcsin\left(\frac{x}{\sqrt{2}}\right) + C \quad \#$$

$$\circledast \quad \boxed{\begin{aligned} \int \frac{1}{\sqrt{a^2-u^2}}du &= \arcsin\left(\frac{u}{a}\right) + C \\ \int \frac{1}{a^2+u^2}du &= \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C \end{aligned}}$$

Ex

$$\int \frac{1}{x\sqrt{5-(\ln x)^2}}\,dx \qquad u = \ln x$$

$$\int e^{\operatorname{cosec} x} \operatorname{cosec} x \cot x \, dx \qquad u = \operatorname{cosec} x$$

$$du = -\operatorname{cosec} x \cot x \, dx$$

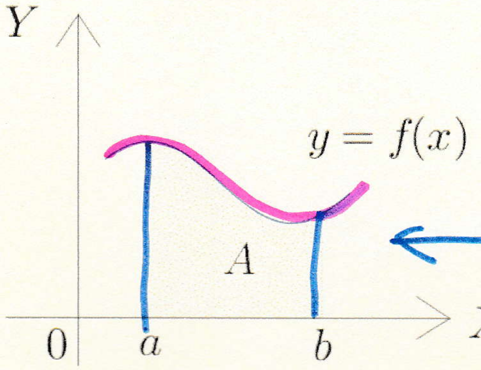
$$\int \frac{5^{\arctan x}}{1+x^2}\,dx \qquad u = \arctan x$$

$$\int \frac{\operatorname{cosec}(\ln x)}{x}\,dx \qquad u = \ln x$$

$$\int \frac{\operatorname{cosec}(\ln x)\cot(\ln x)}{x}\,dx \qquad u = \ln x$$

5.6 The Fundamental Theorem of Calculus

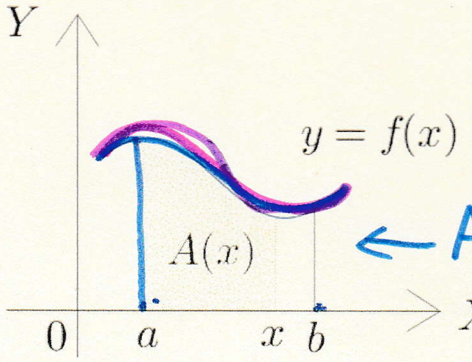
5.6.1 The Fundamental Theorem of Calculus



$A = \int_a^b f(x)dx$ ①

Figure 5.9: area under the graph

(Figure 5.9). Recall that our discussion of the antiderivative method in Section 5.1 suggested that if



$A(x) = \int_a^x f(t)dt$ ②

$A'(x) = f(x)$

Figure 5.10: area under the graph from a to x

$A(x)$ is the area under the graph of f from a to x (Figure 5.10), then

- $A'(x) = f(x)$
- $A(a) = 0$ (The area under the curve from a to a is the area above the single point a, and hence is zero.)
- $A(b) = A$ (The area under the curve from a to b is A.)

Theorem 5.9 (The Fundamental Theorem of Calculus, Part 1) If f is continuous on $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\boxed{\int_a^b f(x)dx} = F(b) - F(a)$$

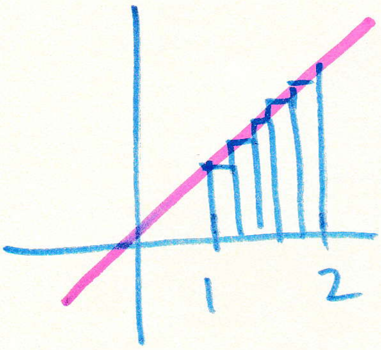
$f(x)$ $\quad F(x) = A(x) + C$

$$\boxed{\int_0^b f(x)dx} = F(b) - F(a)$$

$$= [A(b) + \cancel{C}] - [A(a) + \cancel{C}]$$

$$= \boxed{A}$$

Example 5.15 Evaluate $\int_1^2 x dx$.



$$\int x dx = \underbrace{\frac{x^2}{2}}_{F(x)} + C$$

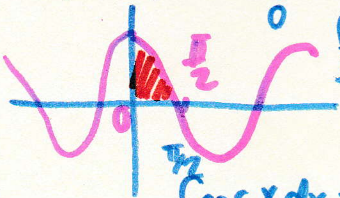
$$\therefore \int_1^2 x dx = F(2) - F(1)$$

$$= \left(\frac{2^2}{2} + \cancel{C}\right) - \left(\frac{1^2}{2} + \cancel{C}\right) = \frac{4}{2} - \frac{1}{2} = \frac{3}{2} \ \#$$

Example 5.16

(a) Find the area under the curve $y = cosx$ over the interval $[0, \pi/2]$. $\quad A = \int_0^{\pi/2} \cos x \, dx$

$$\int \cos x \, dx = \sin x + C$$



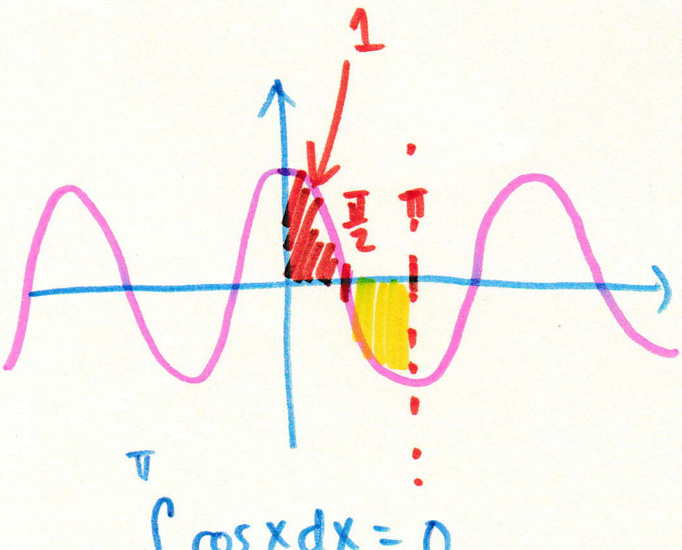
$$\int_0^{\pi/2} \cos x \, dx = \sin\left(\frac{\pi}{2}\right) - \sin(0)$$

$$= 1 - 0 = 1 \ \#$$

(b) Make a conjecture about the value of the integral

$$\int_0^\pi \cos x dx$$

and confirm your conjecture using the Fundamental Theorem of Calculus.



$$\int_0^\pi \cos x \, dx = 0$$

$$\int_0^\pi \cos x \, dx = \sin x \Big]_{x=0}^{\pi}$$

← ขอบเขต ที่จะแทน

$$= \sin(\pi) - \sin(0)$$

$$= 0 - 0 = 0$$

5.6.2 The Relationship Between Definite and Indefinite Integrals

Let F be any antiderivative of the integrand on $[a, b]$, and let C be any constant, then

$$\boxed{\int_a^b f(x)dx = [F(b) + C] - [F(a) - C] = F(b) - F(a)}$$

Thus, for purposes of evaluating a definite integral we can omit the constant of integration in

$$\int_a^b f(x)dx = [F(x) + C]_a^b = \left[\int f(x)dx\right]_a^b$$

which relates the definite and indefinite integrals.

Example 5.17

(a) $\int_4^9 x^2\sqrt{x}dx = \left[\int x^2\sqrt{x}\,dx\right]_4^9 = \left[\int x^{5/2}\,dx\right]_4^9 = \frac{2x^{\frac{7}{2}}}{7}\Big|_{x=4}^{9} = \frac{2(9)^{\frac{7}{2}}}{7} - \frac{2(4)^{\frac{7}{2}}}{7}$

$= \frac{4118}{7} = 588\frac{2}{7}$ #

(b) $\int_0^{\ln 3} 5e^x dx =$

(c) $\int_{-1/2}^{1/2} \frac{1}{\sqrt{1-x^2}}dx =$

(b) $\int_0^{\ln 3} 5e^x dx = 5\int_0^{\ln 3} e^x dx = 5e^x\Big|_{x=0}^{\ln 3}$

$= 5[e^{\ln 3} - e^0] = 5[3-1] = 10$

5.6.3 Part 2 of The Fundamental Theorem of Calculus

Theorem 5.10 If f is continuous on an interval, then f has an antiderivative on that interval. In particular, if a is any point in the interval, then the function F defined by

$$F(x) = \int_a^x f(\underbrace{t}_{}) d\underbrace{t}_{}$$

→ dummy variable

อยากได้ว่า $F'(x) = f(x)$

is an antiderivative of f ; that is, $F'(x) = f(x)$ for each x in the interval, or in an alternative notation

$$\frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$$

Example 5.18 Find $\frac{d}{dx}\left[\int_1^x t^3 dt\right]$

$$\frac{d}{dx}\left[\int_1^x t^3 dt\right] = \frac{d}{dx}\left[\frac{t^4}{4}\Big|_1^x\right]$$

$$= \frac{d}{dx}\left[\frac{x^4}{4} - \frac{1}{4}\right]$$

$$= \frac{4x^3}{4} + 0 = x^3$$

FTC2 $\quad \frac{d}{dx}\left[\int_1^x t^3 dt\right] = x^3$

$$\frac{d}{dx}\int_1^x e^{t}\cos t \, dt = e^{x}\cos x$$

$f(x) = \frac{\sin x}{x}$ จะ $\frac{d}{dx}\left[\int_1^x \frac{\sin t}{t} dt\right] = \frac{\sin x}{x}$ #