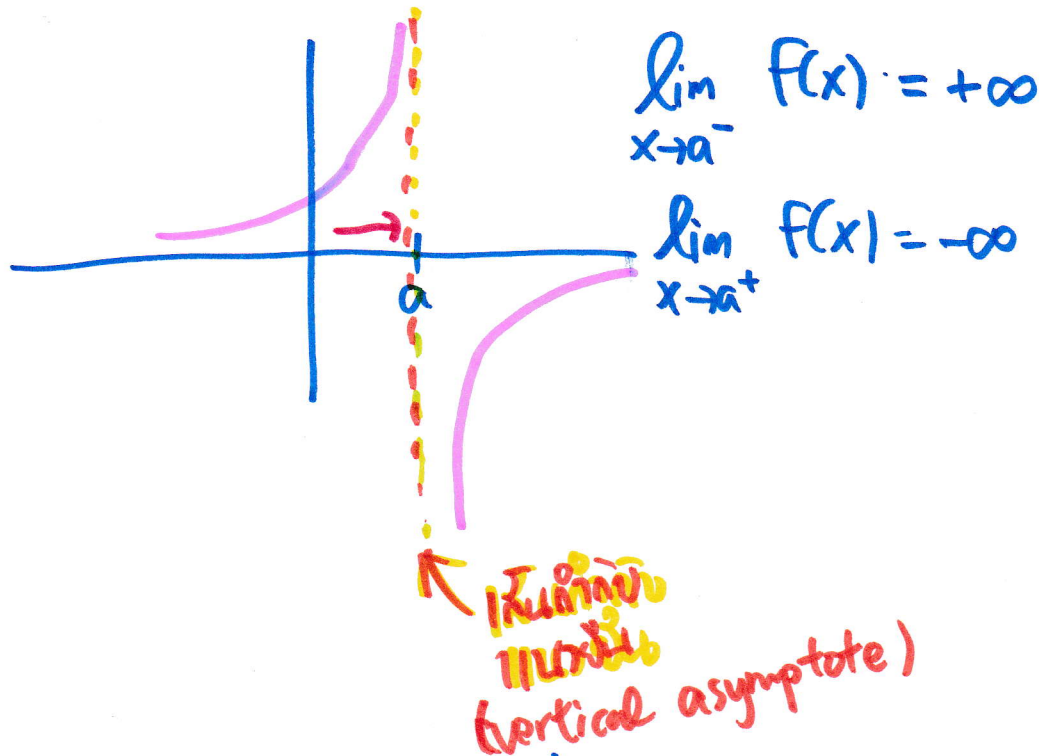
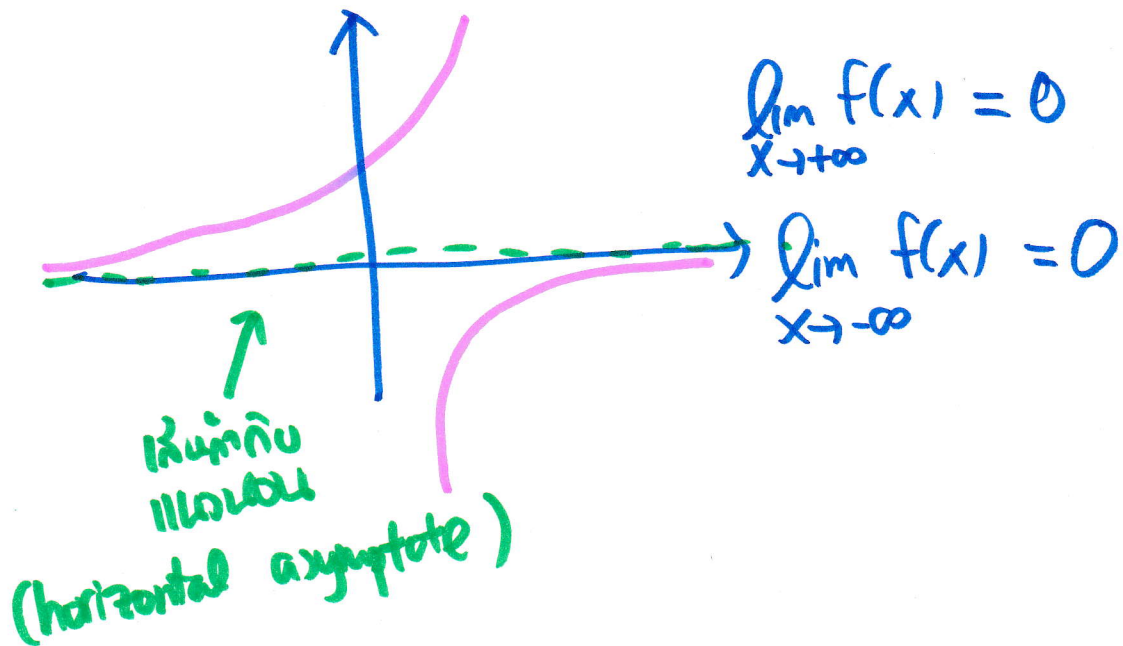


- Limit

- Infinity limit (အကဲခတ်ခြင်း)



- Limit at infinity (အကဲခတ်ခြင်း)



แบบทดสอบย่อย เรื่องลิมิต

ชื่อ-สกุล..... รหัสนักศึกษา..... ลำดับที่.....

1. กำหนดกราฟของฟังก์ชัน f ดังภาพ จงหาลิมิตต่อไปนี้

(a) $\lim_{x \rightarrow -2^+} f(x) = -\infty$

(b) $\lim_{x \rightarrow 0^-} f(x) = 0$

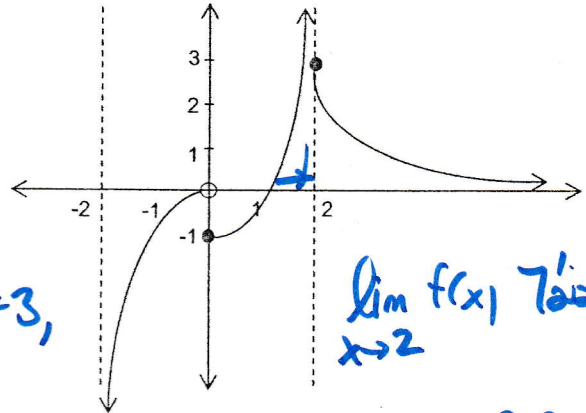
(c) $\lim_{x \rightarrow 0^+} f(x) = -1$

(d) $\lim_{x \rightarrow 2^-} f(x) = +\infty$

(e) $\lim_{x \rightarrow 2^+} f(x) = 3$ $f(2)=3,$

(f) $\lim_{x \rightarrow +\infty} f(x) = 0$

(g) The vertical asymptotes of the graph of f (เส้นกำกับแนวตั้งของกราฟ f) is $x = -2, 2$



2. กำหนดกราฟของฟังก์ชัน f ดังภาพ จงหาลิมิตต่อไปนี้

(a) $\lim_{x \rightarrow 1^-} f(x) = 4$

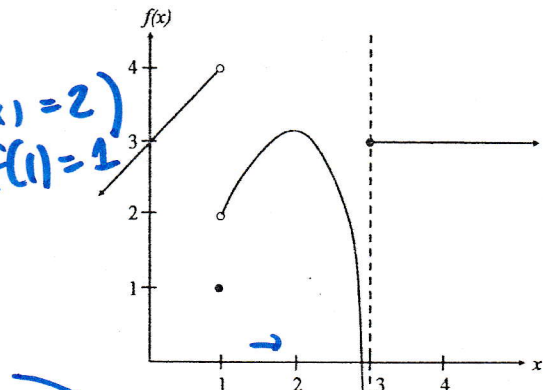
(b) $\lim_{x \rightarrow 1} f(x) = 3$ (เพราะ $\lim_{x \rightarrow 1^-} f(x) = 2$ และ $f(1) = 3$)

(c) $\lim_{x \rightarrow 2} f(x) = 3$

(d) $\lim_{x \rightarrow 3^-} f(x) = -\infty$

(e) $\lim_{x \rightarrow \infty} f(x) = 3$

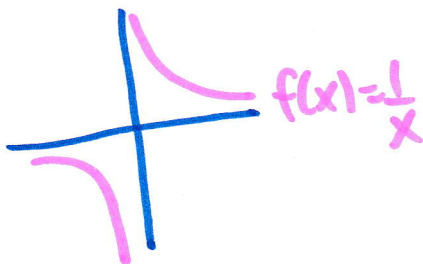
(f) $\lim_{x \rightarrow 4} [x + (f(x))^2] = 13$



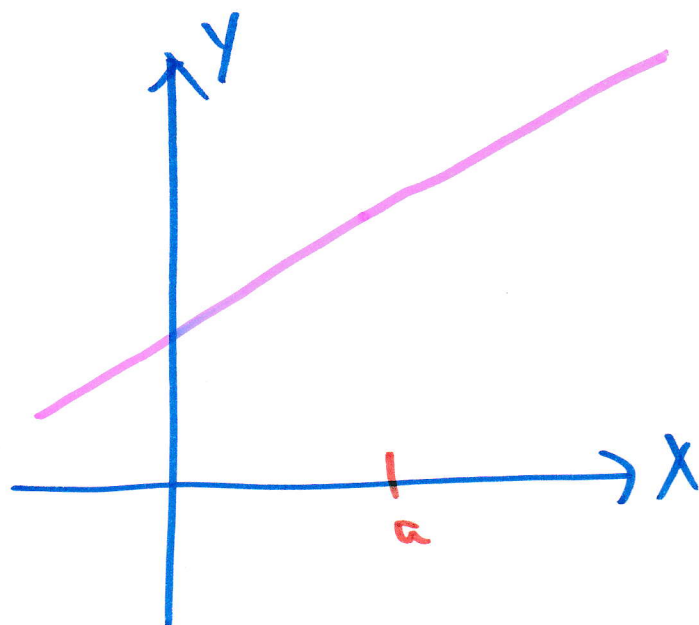
3. จงหาลิมิตต่อไปนี้ $\lim_{x \rightarrow 0^+} \frac{(x+3)(x-2)}{x(x-0.001)} = +\infty$

$\lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty$

$\lim_{x \rightarrow 4} f(x) = 3 \rightarrow \lim_{x \rightarrow 4} (f(x))^2 = 9$
 $\lim_{x \rightarrow 4} x = 4$
 $\therefore \lim_{x \rightarrow 4} [x + (f(x))^2] = 13$



→ အကဲဖြတ်စစ်ချက်များ $x=a$



f မပြတ်ကပ်ဆက်တိုက်မှုရှိ $x=a$



(1) $f(a)$ ရှိမရှိ

(2) $\lim_{x \rightarrow a} f(x)$ ရှိမရှိ

(3) $\lim_{x \rightarrow a} f(x) = f(a)$

① $f(x) = x^2$

② $f(x) = x^2 + 4x$

$f'(x) = 2x$
meaning?

$f'(x) = 2x + 4$

③ $f(x) = x \cdot (x^2 + 1)^2$

$f'(x) = ?$

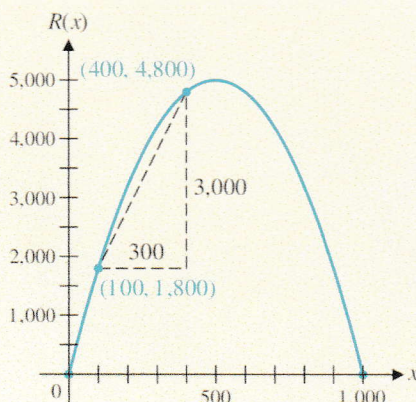
Barnett/Ziegler/Byleen College Mathematics 12th edition with modifications.

๑๕ ลว

רבא לא חזי ליה

A hand-drawn graph illustrating the definition of a function $f(x)$. The x-axis is labeled with x and $x+h$, with a horizontal segment between them labeled h . The y-axis is labeled with $f(x)$ and $f(x+h)$. A pink curve represents the function $y = f(x)$. Dashed vertical lines connect the points $(x, f(x))$ and $(x+h, f(x+h))$ on the curve to the x-axis.

$$\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$



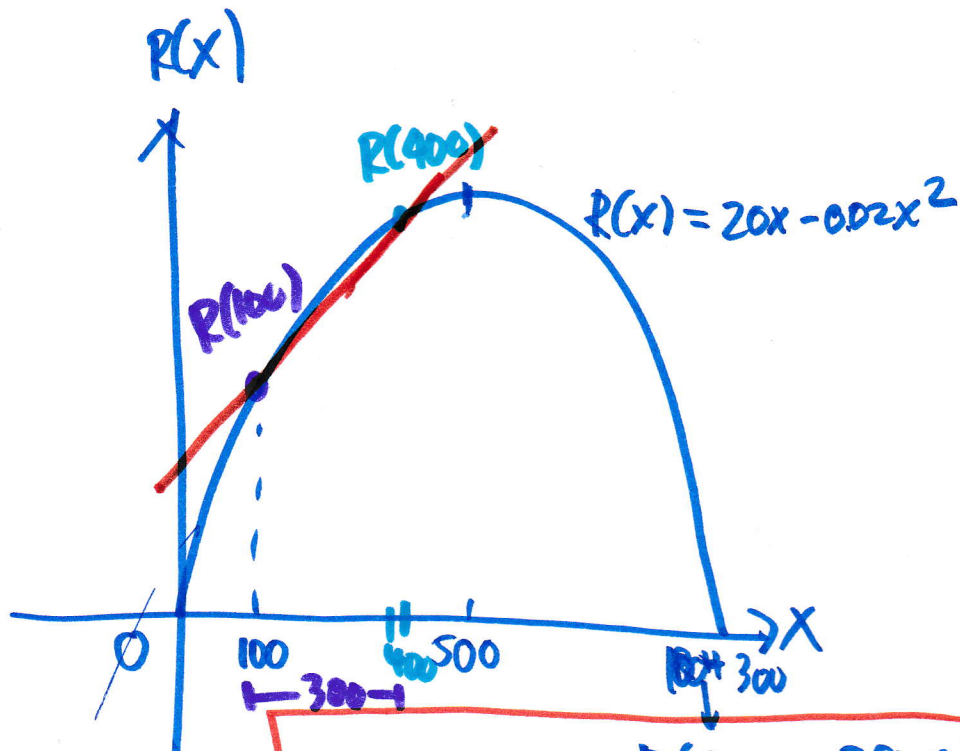
5767

- $$(RGX) : \$$$

$$R(400) - R(100) = [20(400) - (0.02)(400)^2] - [20(100) - (0.02)(100)^2] = 3000$$

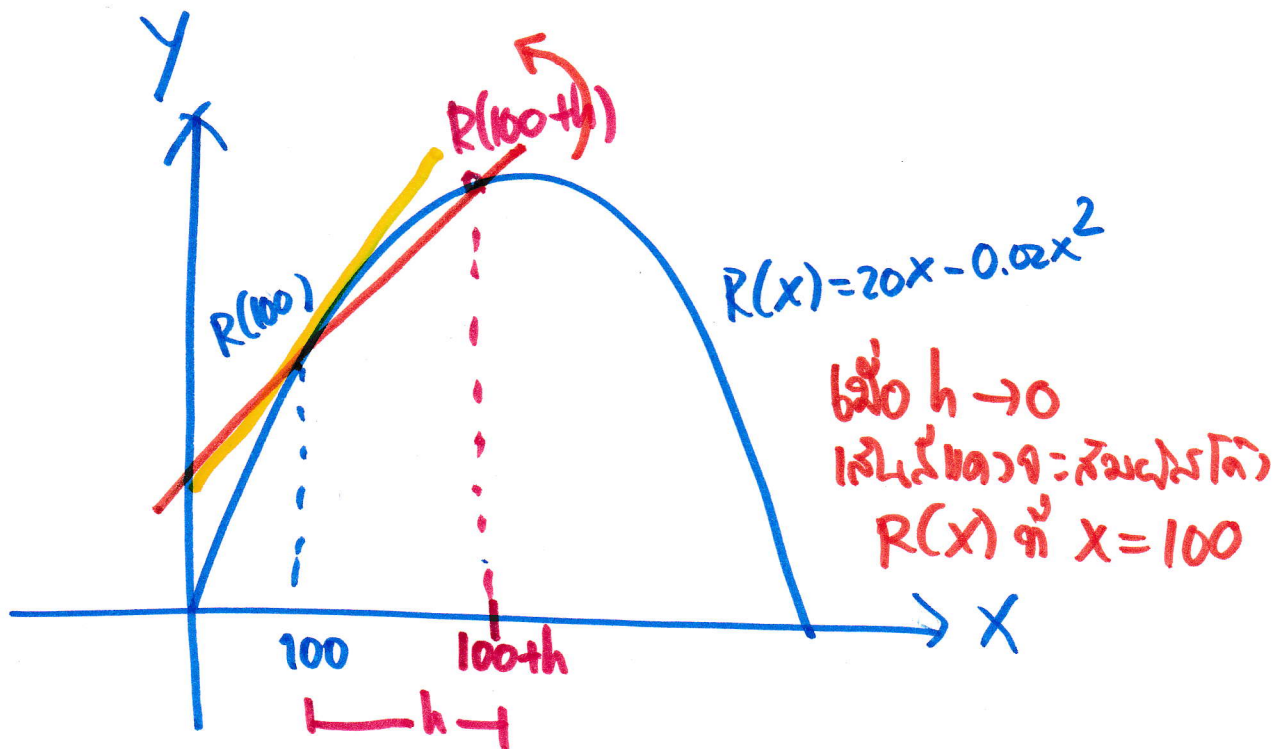
2. What is the average rate of change in revenue (per unit change in x) if production changes from 100 to 400?

$$\frac{R(400) - R(100)}{400 - 100} = \frac{3,000}{300} = 10 \text{ \$/bu}$$



Average
 Rate of
 Change

$$= \frac{R(400) - R(100)}{400 - 100}$$



as $h \rightarrow 0$
 the secant line becomes the tangent line
 $R(x)$ at $x = 100$

take ~~limit~~
 limit as $h \rightarrow 0$

$$R(x) = 20x - 0.02x^2 \quad ; \quad 0 \leq x \leq 1000$$

(10000 + 200h + h²)

3. Find and simplify the average rate of change from $x = 100$ to $x = 100 + h$, $h \neq 0$.

$$\begin{aligned} \frac{R(100+h) - R(100)}{(100+h) - 100} &= \frac{[(20(100+h) - (0.02)(100+h)^2)] - [(20)(100) - (0.02)(100)^2]}{100+h-100} \\ &= \frac{[2000 + 20h - (200 + 40h + 0.02h^2)] - [2000 - 200]}{h} \\ &= \frac{20h - 40h - 0.02h^2}{h} = 20 - 4h - 0.02h \end{aligned}$$

4. Find the limit of the expression from part (3) as $h \rightarrow 0$ if that limit exists and discuss possible interpretations of the limit.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{R(100+h) - R(100)}{h} &= \lim_{h \rightarrow 0} (20 - 4h - 0.02h) \\ &= 20 \end{aligned}$$

$$\begin{aligned} &= \frac{20h - 4h - 0.02h^2}{h} \\ &= 20 - 4h - 0.02h \end{aligned}$$

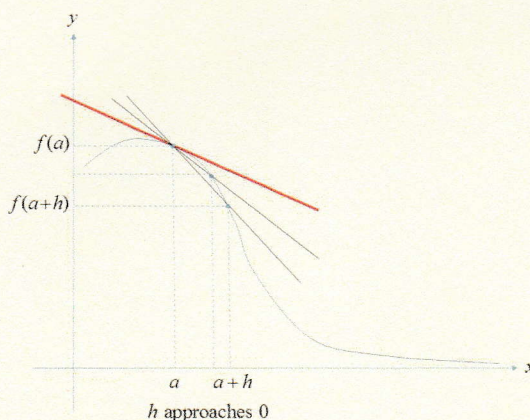
5.2 Instantaneous Rate of Change

For $y = f(x)$, the instantaneous rate of change at $x = a$ is

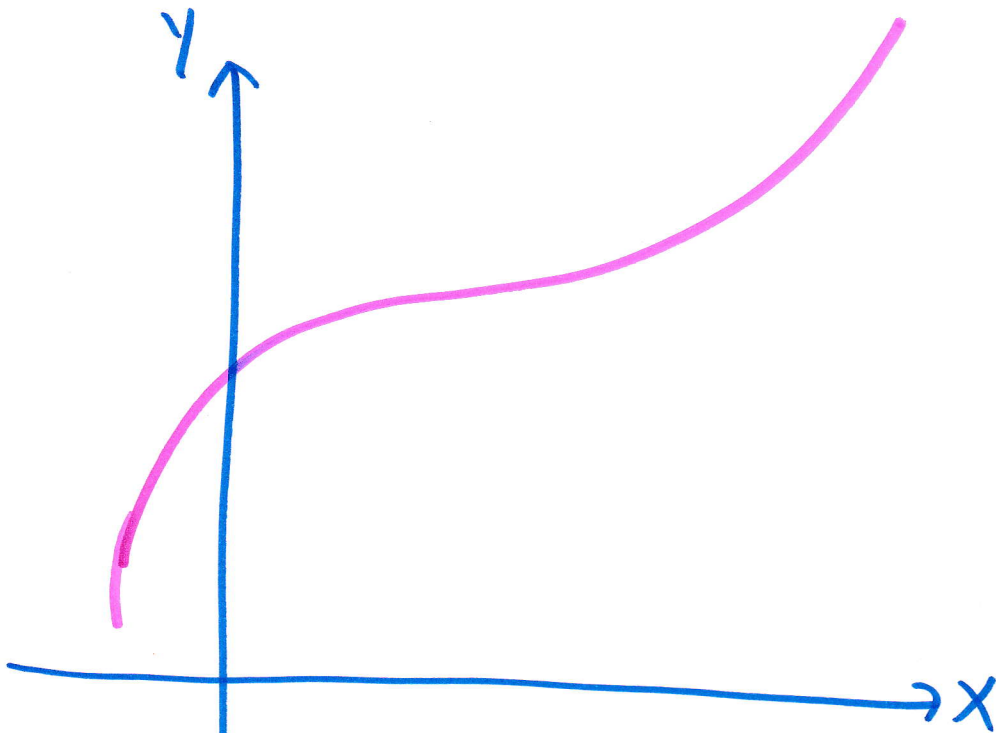
$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

if the limit exists.

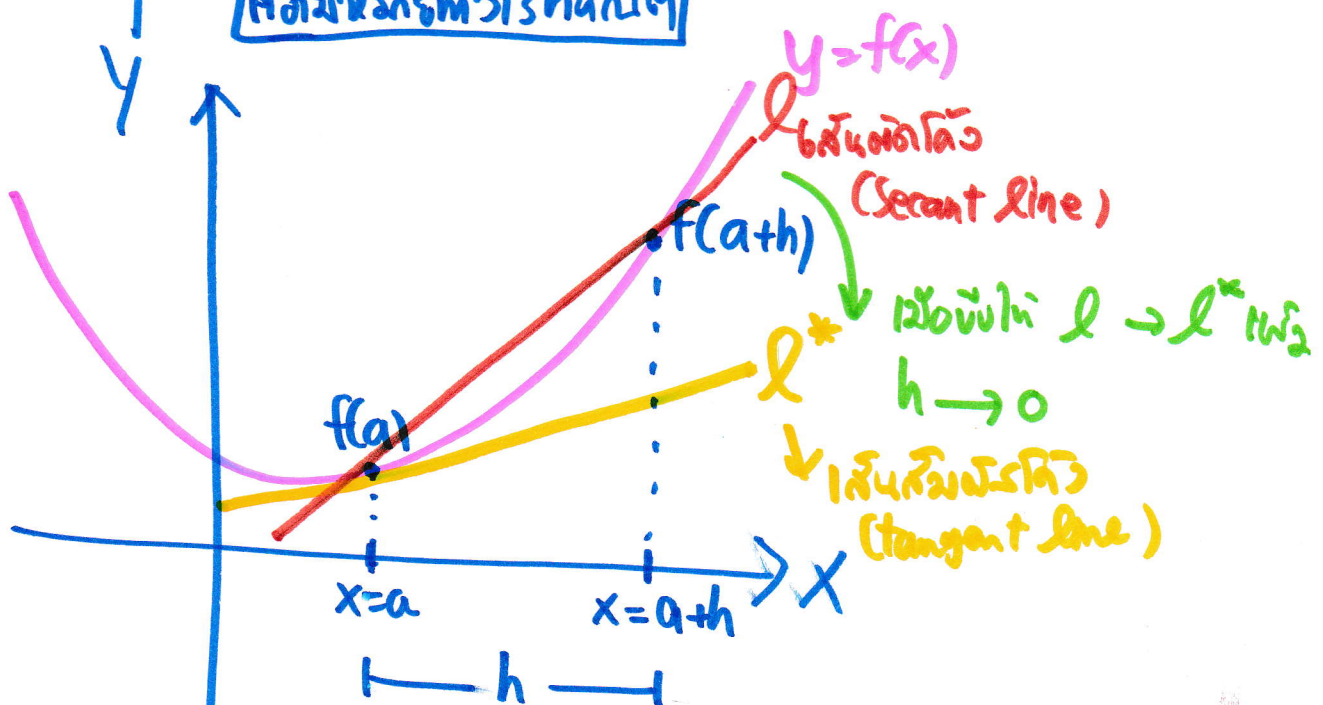
Instantaneous rate of change = slope of the tangent line



ถ้า $y=f(x)$ เป็นฟังก์ชัน.



อัตราส่วนเฉลี่ย



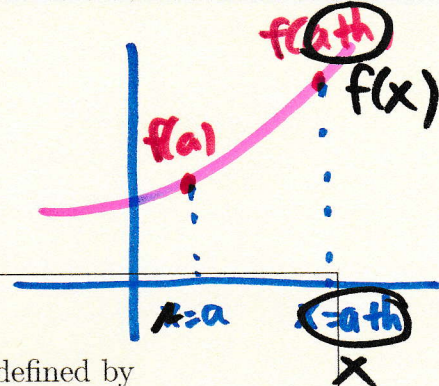
อัตราส่วนของเส้นเชื่อมสองจุด, $l = \frac{f(a+h) - f(a)}{(a+h) - a}$
 $= \frac{f(a+h) - f(a)}{h}$

อัตราส่วนของเส้นสัมผัสกับ $f(x)$ ที่ $x=a$

Slope = $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$
 ความชัน
 ของเส้นสัมผัสกับ $f(x)$ ที่ $x=a$

อัตราการเปลี่ยนแปลง
 ณ จุด $x=a$
 Instantaneous rate of change

5.3 The Derivative



Let $f(x)$ be a function.

1. The **derivative** of f at x , denoted by $f'(x)$, is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

2. The derivative of f at $x = a$, denoted by $f'(a)$, is defined by

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Handwritten notes: $h \rightarrow 0 \Rightarrow x \rightarrow a$, $u = a+h = x$

If $f'(a)$ exists (the limit exists), then f is said to be differentiable at a . If $f'(x)$ exists (the limit exists) for each x in the open interval (a, b) , then f is said to be differentiable over (a, b) .

Given $y = f(x)$, the derivative of f at x may be represented by any of the followings:

$$f'(x)$$

$$y'$$

$$\frac{dy}{dx}$$

$$\frac{df}{dx}$$

$$f(x) = 4x^2 + 12x + 9$$

Example 5.3.1. Given $f(x) = (2x + 3)^2$. Use the definition of the derivative to find the followings.

$$1. f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{[2(x+h)+3]^2 - (2x+3)^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(2(x+h))^2 + 2 \cdot (2(x+h))(3) + 3^2] - [4x^2 + 12x + 9]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[4(x^2 + 2xh + h^2) + 12x + 12h + 9] - [4x^2 + 12x + 9]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[4x^2 + 8xh + 4h^2 + 12x + 12h + 9] - [4x^2 + 12x + 9]}{h}$$

3. The equation of the tangent line of $y = f(x)$ at $x = 2$

$$= \lim_{h \rightarrow 0} \frac{8xh + 4h^2 + 12h}{h}$$

$$= \lim_{h \rightarrow 0} 8x + 4h + 12$$

$$= 12 + 8x$$

$$\therefore f'(x) = 8x + 12$$

$$f'(a) = 8(2) + 12 = 28$$

$$f'(2) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2}$$

$$2. f'(2)$$

สมการเส้นสัมผัส (x, y), จุดบน M
เส้นสัมผัสเส้นโค้งที่จุด
 $y - y_1 = m(x - x_1)$

ถ้า $x = 2$ แล้ว $f(2) = (2 \cdot 2 + 3)^2 = 49$
จุดบนเส้นโค้ง (2, 49)

หาค่า m: $m = f'(2) = 28$

$\therefore$ สมการเส้นสัมผัสคือ $y - 49 = (28)(x - 2)$



การบ้าน (HW) (ส่งด้วย)

① ให้ $f(x) = \sqrt{x}$ (โปรด)

(1) จงหา $f'(x)$ โดยใช้กฎอนุพันธ์

(2) จงหา $f'(1)$

(3) จงหาสมการเส้นสัมผัสกราฟ $f(x) = \sqrt{x}$ ที่ $x = 4$.

② (Optional)

ให้ $f(x) = \frac{1}{2x-4}$

(1) จงหา $f'(x)$ โดยใช้กฎอนุพันธ์

(2) จงหา $f'(3)$

(3) จงหาสมการเส้นสัมผัสกราฟ $f(x) = \frac{1}{2x-4}$ ที่

(3.1) $x = 3$

(3.2) $x = 1$