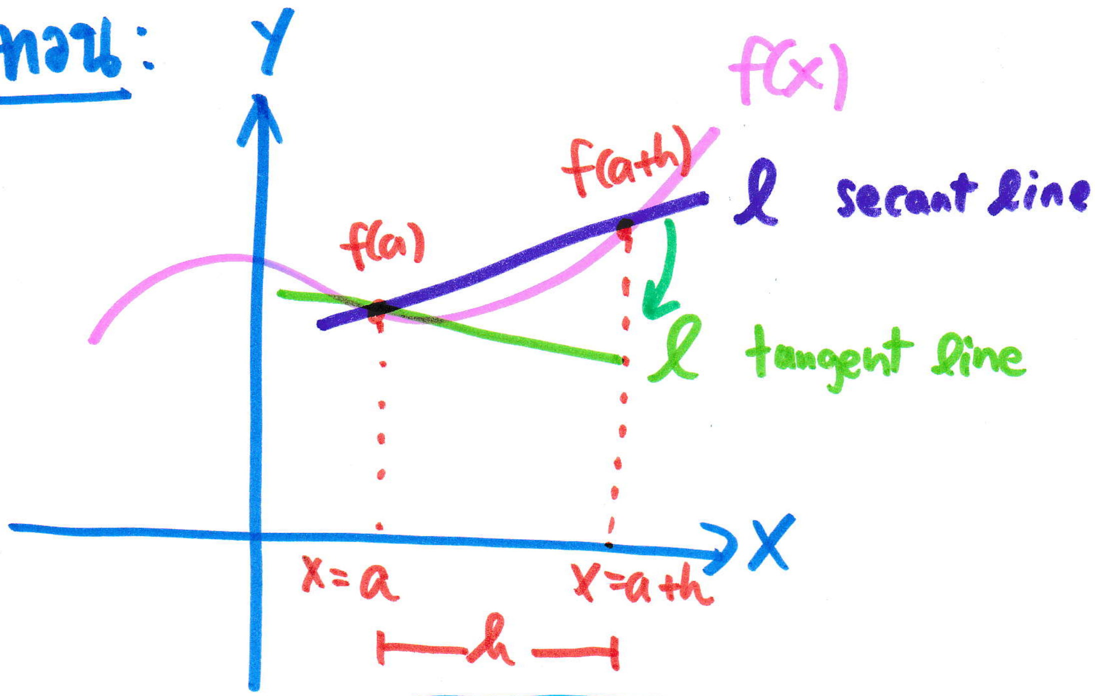
## ทบทวน:



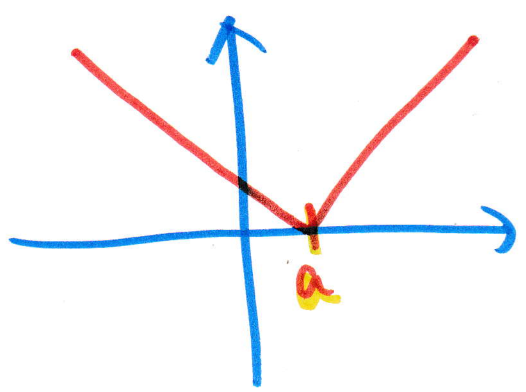

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}} \quad \text{—(1)}$$

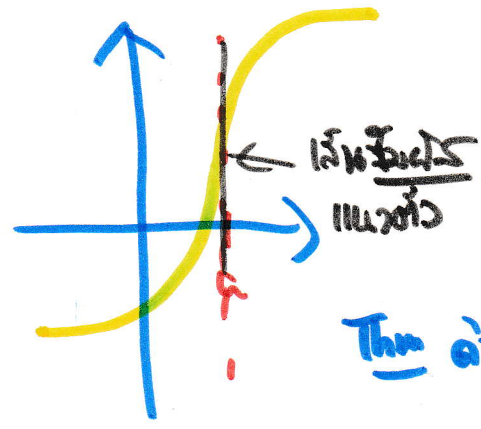
$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}} \quad \text{—(2)}$$

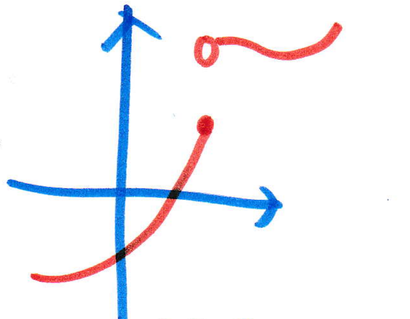
ตรวจสอบว่า $f'(a)$ มีไหม (differentiable ไหม)

(1) $\lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h} \to$ มี

(2) $\lim_{x \to a^-} \frac{f(x) - f(a)}{x-a} = \lim_{x \to a^+} \frac{f(x) - f(a)}{x-a} \to$ มี.



Thm ถ้า f diff ได้แล้ว f ต่อเนื่อง

≡ ถ้า f ไม่ต่อเนื่องแล้ว f diff ได้

# แบบทดสอบย่อย เรื่องนิยามของอนุพันธ์

ชื่อ-สกุล.................................................................. รหัสนักศึกษา........................... ลำดับที่...........

1. ให้ $f$ เป็นฟังก์ชัน กำหนดโดย

$$f(x) = \begin{cases} \boxed{3x+2} & ; x < 1 \\ \boxed{1-x^2} & ; x \geq 1 \end{cases}$$

(a) จงหา $\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1}$ $= \lim_{x\to 1^-} \frac{3x+2-0}{x-1}$

$= \lim_{x\to 1^-} \frac{3x+2}{x-1} = -\infty$

(b) จงหา $\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$ $= \lim_{x\to 1^+} \frac{1-x^2-0}{x-1}$

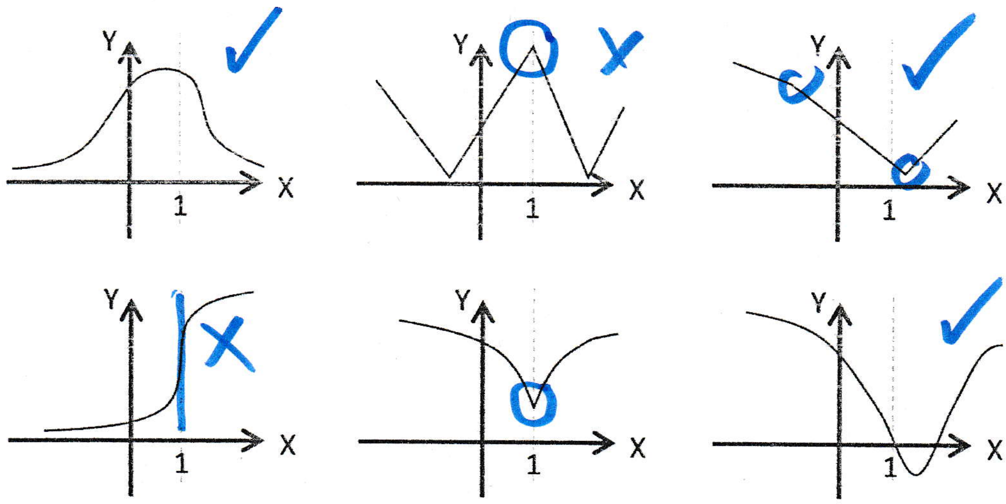
$= \lim_{x\to 1^+} \frac{\overset{-1}{\cancel{(1-x)}}(1+x)}{\cancel{(x-1)}}$

$= \lim_{x\to 1^+} -(1+x) = -2$

(c) $f$ หาอนุพันธ์ได้ที่ $x = 1$ หรือไม่ เพราะเหตุใด

f หาอนุพันธ์ไม่ได้ที่ $x=1$ เพราะ $\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} \neq \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$

2. กราฟใดต่อไปนี้หาอนุพันธ์ได้ที่ $x = 1$



HW [illegible] $\dfrac{d(c)}{dx}=0,\ \dfrac{dx^n}{dx}=nx^{n-1}$

**Example 5.4.5.** Find

1. $g'(t)$ for $g(t) = 5t^{1/5} - \dfrac{8}{t^{3/2}}$ $\rightarrow 8t^{-3/2}$

$g'(t) = \cancel{5}(\frac{1}{\cancel{5}})t^{\frac{1}{5}-1} - \overset{4}{\cancel{8}}(-\frac{3}{\cancel{2}_1})t^{-\frac{3}{2}-1}$

$y' = t^{-\frac{4}{5}} + 12t^{-5/2} = \dfrac{1}{t^{4/5}} + \dfrac{12}{t^{5/2}}$ #

2. $\dfrac{dy}{dw}$ for $y = \dfrac{5}{w^6} - 2\sqrt{w}$

$\dfrac{dy}{dw} = 5w^{-6} - 2w^{\frac{1}{2}}$

$= (5)(-6)w^{-6-1} - 2(\frac{1}{2})w^{\frac{1}{2}-1}$

$= -30w^{-7} - w^{-\frac{1}{2}} = -\dfrac{30}{w^7} - \dfrac{1}{\sqrt{w}}$

$\boxed{\dfrac{d}{dx}(\sqrt{x}) = \dfrac{1}{2\sqrt{x}}}$

**Example 5.4.6.** Let $f'(2) = 3$ and $g'(2) = -1$. Find $h'(2)$ for $h(x) = -4f(x) + 5g(x) - 9$.

$h'(x) = -4f'(x) + 5g'(x)$

$\therefore h'(2) = -4f'(2) + 5g'(2)$

$= -12 + (5)(-1) = -17.$

**Applications**: Tangent line slope and equation at a point on the curve of the function

**Example 5.4.7.** Let $f(x) = x^4 - 6x^2 + 10$.

1. Find $f'(x)$. $= 4x^3 - 12x$

2. Find the equation of the tangent line at $x = 1$.

หา $f'(1) = 4 - 12 = \boxed{-8}$

จุดสัมผัส $x = 1$

$f(1) = 1 - 6 + 10 = 5$

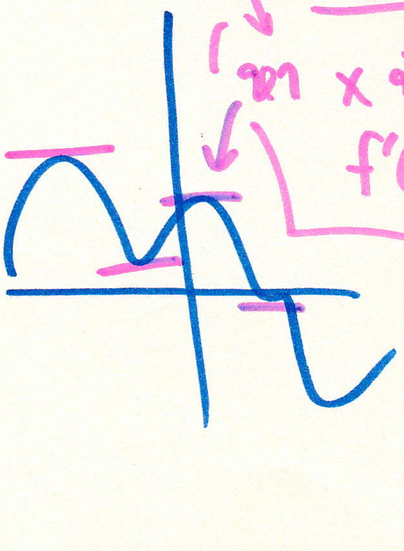
จุดสัมผัส $(1,5)$

สมการ $\boxed{y - 5 = (-8)(x-1)}$

3. Find the values of $x$ where the tangent line is horizontal.

เส้นสัมผัสขนานแกน x slope = 0

หา x ที่ทำให้ $f'(x) = 0$



$\Rightarrow 4x^3 - 12x = 0$

$x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$x(x-\sqrt{3})(x+\sqrt{3}) = 0$

$x = 0, -\sqrt{3}, \sqrt{3}$ #

$\boxed{y=f(x)}$ $\quad f'(x) \quad \frac{dy}{dx} \quad \frac{df}{dx}$

$y'$

## 5.5 Derivatives of products and quotients

**Derivatives of Products**

Let $y = f(x) = F(x)S(x)$. If $F'(x)$ and $S'(x)$ exist, then

$$f'(x) = F(x)\frac{d}{dx}[S(x)] + S(x)\frac{d}{dx}[F(x)].$$

F.= น.ล

$\boxed{F' = \text{นล}' + \text{ลน}'}$

**Quotient Rule**

Let $y = f(x) = \dfrac{T(x)}{B(x)}$. If $T'(x), B'(x)$ exist and $B(x) \neq 0$, then

$$f'(x) = \frac{B(x)\frac{d}{dx}[T(x)] - T(x)\frac{d}{dx}[B(x)]}{[B(x)]^2}.$$

$F = \dfrac{\text{บ}}{\text{ล}}$

$\boxed{F' = \dfrac{\text{ลบ}' - \text{บล}'}{\text{ล}^2}}$

**Example 5.5.1.** Find $\boxed{\dfrac{df}{dx}}$ for $f(x) = \overset{\text{น}}{\boxed{2x^2}}\overset{\text{ล}}{\boxed{(3x^4 - 2)}}$.

$f'(x) = (2x^2)(12x) + (3x^4-2)(4x)$ #

$f'(x) = 2x^2\frac{d}{dx}(3x^4-2) + (3x^4-2)\frac{d}{dx}(2x^2)$

**Example 5.5.2.** Consider $\boxed{f(x)} = \overset{\text{น.}}{\boxed{(x^2 - 4x)}}\overset{\text{ล.}}{\boxed{(2x^2 - x + 3)}}$. Find $f'(x)$ and $f'(1)$.

$f'(x) = (x^2-4x)\frac{d}{dx}(2x^2-x+3) + (2x^2-x+3)\frac{d}{dx}(x^2-4x)$

$= (x^2-4x)(4x-1) + (2x^2-x+3)(2x-4)$ #

$f'(1) = (1-4)(4-1) + (2-1+3)(2-4)$

$= (-3)(3) + (4)(-2) = -9-8 = -17$ #

**Example 5.5.3.** Find $\dfrac{dy}{dx}$

1. $y = \dfrac{\boxed{x^2 - 1}}{\boxed{x^2 + 1}}$ บ ล

$\dfrac{dy}{dx} = \dfrac{(x^2+1)\frac{d}{dx}(x^2-1) - (x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$

$= \dfrac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$

$= \dfrac{\cancel{2x^3} + 2x - \cancel{2x^3} + 2x}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$ #

2. $y = \dfrac{(x-2)(x^2+1)}{x^5}$

$$\frac{dy}{dx} = \frac{x^5\frac{d}{dx}[(x-2)(x^2+1)] - (x-2)(x^2+1)\frac{d}{dx}(x^5)}{(x^5)^2}$$

$$= \frac{x^5\left[(x-2)\frac{d}{dx}(x^2+1) + (x^2+1)\frac{d}{dx}(x-2)\right] - (x-2)(x^2+1)(5x^4)}{x^{10}}$$

$$= \frac{x^5[(x-2)(2x) + (x^2+1)(1)] - (x-2)(x^2+1)(5x^4)}{x^{10}}$$

#

## 5.6 Derivatives of Exponential Functions, Logarithmic Functions and Trigonometric Functions

$f(x) = 2^x$ VS $f(x) = x^2$

[illegible] ~~$f'(x) = x2^{x-1}$~~ $\qquad f'(x) = 2x$

**Definition: Value of** $e$

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.718281828459\ldots$$

เลือก $a = e$

$\frac{d}{dx}[e^x] = e^x \ln e = e^x$

| | |
|---|---|
| 1. $\dfrac{d}{dx}[a^x] = a^x \ln a$ | 2. $\dfrac{d}{dx}[e^x] = e^x$ |
| 3. $\dfrac{d}{dx}[\log_a x] = \dfrac{1}{x\ln a}$ | 4. $\dfrac{d}{dx}[\ln x] = \dfrac{1}{x}$ |

$\ln x = \log_e x \Rightarrow \frac{d}{dx}[\log_e x] = \frac{d}{dx}[\ln x] = \frac{1}{x\ln e} = \frac{1}{x}$

**Example 5.6.1.** Find $y'$ for

1. $y = 5\ln x$

(1) $y' = 5\frac{d}{dx}(\ln x) = 5\cdot\frac{1}{x} = \frac{5}{x}$ #

2. $y = -7x^e - 2e^x + e^2$

$\rightarrow y' = (-7)ex^{e-1} - 2e^x + \cancel{2e}^{\,0}$

$= -7ex^{e-1} - 2e^x$ #

3. $y = 2^x - 3^x + \log_5 x^4$

$y' = 2^x\ln 2 - 3^x\ln 3 + \frac{1}{x^4\ln 5}\frac{d}{dx}(x^4)$

โจท $y = 2e^x + \ln(x) + \ln(5) - \ln(e)$

$y' = 2e^x + \frac{1}{x} + 0 + 0$

**Derivatives of Trigonometric Functions**

| | |
|---|---|
| $\frac{d}{dx}[\sin x] = \cos x$ | $\frac{d}{dx}[\cos x] = -\sin x$ |
| $\frac{d}{dx}[\tan x] = \sec^2 x$ | $\frac{d}{dx}[\cot x] = -\csc^2 x$ |
| $\frac{d}{dx}[\sec x] = \sec x \tan x$ | $\frac{d}{dx}[\csc x] = -\csc x \cot x$ |

**Example 5.6.2.** Find the derivatives of the following functions

1. $y = 5x + 2\sin x$

$$y' = 5 + 2\frac{d}{dx}(\sin x)$$
$$= 5 + 2\cos x \quad \#$$

2. $y = t\cot t$

$$y' = t\frac{d}{dt}(\cot t) + \cot t\,\frac{d}{dt}(t)$$
$$= t(-\operatorname{cosec}^2 t) + \cot t$$
$$= -t\operatorname{cosec}^2 t + \cot t \quad \#$$

3. $r = \dfrac{\tan\theta}{1 + \tan\theta}$

$$y' = \frac{(1+\tan\theta)\frac{d}{d\theta}(\tan\theta) - \tan\theta\frac{d}{d\theta}(1+\tan\theta)}{(1+\tan\theta)^2}$$
$$= \frac{(1+\tan\theta)(\sec^2\theta) - (\tan\theta)(\sec^2\theta)}{(1+\tan\theta)^2} \quad \#$$

4. $y = \dfrac{\sin x}{x}$

$$y' = \frac{x\frac{d}{dx}(\sin x) - (\sin x)\frac{dx}{dx}}{x^2}$$
$$= \frac{x\cos x - \sin x}{x^2} \quad \#$$

5. $f(x) = e^x \sin x - 5\cos x$

$$f'(x) = \frac{d}{dx}(e^x \sin x) - 5\frac{d}{dx}(\cos x)$$
$$= \left[e^x\frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}(e^x)\right] + 5(-\sin x)$$
$$= e^x\cos x + e^x \sin x - 5\sin x \quad \#$$

HW ① จงหาอนุพันธ์ของฟังก์ชันต่อไปนี้

(1) หา $y'$ เมื่อ $y = 2\cos x - \log_2 x$

(2) หา $g'(x)$ เมื่อ $g(x) = (x^2-5)e^x$

(3) หา $\frac{dy}{dx}$ เมื่อ $y = \frac{\sin x \cos x}{x^7+7}$

ส่งต้นคาบนะ

② จำสูตรอนุพันธ์ให้ครบ

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- ประเมินอาจารย์ , ประเมินกระบวนวิชา