แบบทดสอบย่อย

เพื่อเช็คชื่อเข้าชั้นเรียน ประจำวันจันทร์ที่ 30 ตุลาคม พ.ศ.2560

ชื่อ-สกุล.. รหัสนักศึกษา............................ ลำดับที่...........

1. จงแสดงวิธีการหาค่าของ $\frac{d}{dx}\left[\int_1^x (1+t^3)^{1/3}\,dt\right]$ FTC (2)

หมายเหตุ ถ้าจะอ้าง Fundamental Theorem of Calculus ให้ระบุเงื่อนไขของการใช้ให้ชัดเจน รวมถึงระบุว่าใช้ข้อไหนด้วย

$f(x) = (1+x^3)^{\frac{1}{3}}$ ต่อเนื่องบน $[1,\infty)$

$$\frac{d}{dx}\left[\int_1^x (1+t^3)^{1/3}\,dt\right] = (1+x^3)^{\frac{1}{3}}$$

2. จงหาค่าของปริพันธ์ $\int_{-1}^{0} \frac{1}{1-3x}\,dx$ $\cancel{\frac{1}{1-3x} = \frac{1}{1} - \frac{1}{3x}}$

let $u = 1-3x$

$du = -3dx$

$u = 1$

$$\int_{-1}^{0} \frac{1}{1-3x}\,dx = -\frac{1}{3}\int_{-1}^{0} \frac{1}{u}\,du$$

$u = 4$

$\int \pi \, d\theta$ ตัวแปร ค่าคงที่

$\int x^n dx$

$\int a^x dx = \frac{a^x}{\ln a} + C$

$$\boxed{\int e^x \cos x \, dx}$$

$$d(uv) = u\,dv + v\,du$$

$$\int d(uv) = \int u\,dv + \int v\,du$$

$$uv = \int u\,dv + \int v\,du$$

$$\therefore \boxed{\int u\,dv = uv - \int v\,du}$$

เลือก u, dv

$u \downarrow du$ $\quad$ $dv \downarrow v + C$

$$\int u\,dv = u(v+c) - \int (v+c)\,du$$

$$= uv + \cancel{uc} - \int v\,du - \cancel{cu}$$

$$= uv - \int v\,du$$

$\int \ln x \, dx$

เลือก $u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x} dx$ $\quad v = \int dx$

$\therefore v = x$

$$\therefore \int \ln x \, dx = x \ln x - \int \cancel{x} \left(\frac{1}{\cancel{x}}\right) dx$$

$$= x \ln x - x + C \quad \#$$

LIATE

L → Logarithm

I → inverse trigone

A → Algebraic f²

T → Trigone

E → exponential

7

Techniques of Integration

7.1 An overview of integration methods

A review of familiar integration formulas

1. $\int du = u + C$	2. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
3. $\int \frac{1}{u}\, du = \ln\lvert u\rvert + C$	4. $\int a^u\, du = \frac{a^u}{\ln a} + C,\ a > 0, a \neq 1$
5. $\int e^u\, du = e^u + C$	
6. $\int \sin u\, du = -\cos u + C$	7. $\int \cos u\, du = \sin u + C$
8. $\int \sec^2 u\, du = \tan u + C$	9. $\int \csc^2 u\, du = -\cot u + C$
10. $\int \sec u \tan u\, du = \sec u + C$	11. $\int \csc u \cot u\, du = -\csc u + C$
12. $\int \tan u\, du = ln\lvert \sec u\rvert + C$	13. $\int \cot u\, du = ln\lvert \sin u\rvert + C$
14. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin(\frac{u}{a}) + C$	15. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan(\frac{u}{a}) + C$

7.2.2 Repeated integration by parts

Example 7.3 Evaluate $\int x^2 e^x \, dx$.

เลือก $u = x^2$ | $dv = e^x dx$

$du = 2x\,dx$ | $v = \int e^x dx$

$v = e^x$

$$\therefore \int x^2 e^x dx = x^2 e^x - \int (e^x)(2x\,dx)$$
$$= x^2 e^x - 2\boxed{\int x e^x dx} \quad \text{---} (*)$$

พิจารณา $\int x e^x dx$

เลือก $u_1 = x$ $\quad dv_1 = e^x dx$

$du_1 = dx$ $\quad v_1 = e^x$

$$\therefore \int x e^x dx = x e^x - \int e^x dx$$
$$\boxed{= x e^x - e^x}$$

$$\therefore \int x^2 e^x dx = x^2 \cdot e^x - 2[x e^x - e^x] + C$$
$$= x^2 e^x - 2x e^x + 2e^x + C \quad \#$$

Example 7.4 Evaluate $\int e^x \cos x \, dx$.

$$\boxed{\int e^x \cos x \, dx}$$

เลือก $u = \cos x \qquad dv = e^x dx$

$du = -\sin x \, dx \qquad v = e^x$

$$\therefore \int e^x \cos x \, dx = e^x \cos x + \boxed{\int e^x \sin x \, dx}$$

พิจารณา $\int e^x \sin x \, dx$

เลือก $u_1 = \sin x \qquad dv_1 = e^x dx$

$du_1 = \cos x \qquad v_1 = e^x$

จะเห็นว่า $\boxed{\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx}$

$$\therefore \underline{\int e^x \cos x \, dx} = e^x \cos x + e^x \sin x - \underline{\int e^x \cos x \, dx}$$

$$2\int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\therefore \int e^x \cos x \, dx = \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C \quad \#$$

7.2.3 Integration by parts for definite integrals

Example 7.5 Evaluate $\int_0^1 \tan^{-1} x \, dx$.

เลือก $u = \arctan x \qquad dv = dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = \int dx = x$

$$\therefore \int_0^1 \arctan x \, dx = x \arctan x \Big]_0^1 - \boxed{\int_0^1 \frac{x}{1+x^2}\,dx} \quad \text{---} (*)$$

พิจารณา $\int_0^1 \frac{x}{1+x^2}\,dx$ เลือก $\heartsuit = 1+x^2$

$d\heartsuit = 2x\,dx$

$x = 0 \Rightarrow \heartsuit = 1+(0)^2 = 1$

$x = 1 \Rightarrow \heartsuit = 1+1^2 = 2$

$$\therefore \int_0^1 \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_1^2 \frac{1}{\heartsuit}\,d\heartsuit$$

$$= \frac{1}{2}\ln|\heartsuit| \Big]_{\heartsuit=1}^2$$

$$= \frac{1}{2}\left[\ln|2| - \cancel{\ln|1|}^{\,0}\right]$$

$$\therefore \int_0^1 \arctan x \, dx = x \arctan x \Big]_0^1 - \frac{1}{2}\ln|\heartsuit| \Big]_{\heartsuit=1}^2$$

$$= \left[1\cdot\arctan 1 - \cancel{0\cdot\arctan 0}^{\,0}\right] - \frac{1}{2}\ln 2$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln 2$$

$$= \frac{\pi}{4} - \ln\sqrt{2}$$

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ปัญหาท้าทายปัญญา Reduction Formula

$$\int \sin^n x \, dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x \, dx$$

$$\int \cos^n x \, dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x \, dx$$

จงพิสูจน์ Reduction Formula

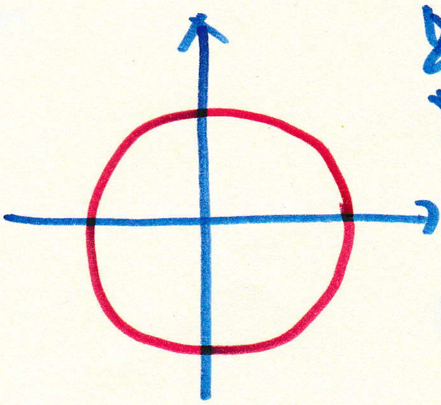
Hint: By parts

โดยแยก $\cos^n x = \underline{\cos^{n-1}x} \cdot \underline{\cos x}$

แล้ว integrate by parts.

โดยเลือก $u = \cos^{n-1}x$; $dv = \cos x \, dx$

จากนั้น ใช้เอกลักษณ์ตรีโกณ $\sin^2 x = 1 - \cos^2 x$



$$\left.\begin{aligned} y &= \sin\theta \\ x &= \cos\theta \end{aligned}\right\} \quad x^2+y^2=1$$

$$\boxed{\sin^2\theta+\cos^2\theta = 1}$$

เอา $\sin^2\theta$ หารตลอด

$$\boxed{1+\cot^2\theta = \operatorname{cosec}^2\theta}$$

เอา $\cos^2\theta$ หารตลอด

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\boxed{\sin(2x) = 2\sin x\cos x}$$

$$\boxed{\cos(2x) = \cos^2 x - \sin^2 x}$$

$$\boxed{\begin{aligned} \sin(A\pm B) &= \sin A\cos B \pm \cos A\sin B \\ \cos(A\pm B) &= \cos A\cos B \mp \sin A\sin B \end{aligned}}$$

$$\sin^2 x = \frac{1}{2}(1-\cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$\int \sin x\,dx = -\cos x + C$$

$$\boxed{\int \sin mx\,dx = -\frac{\cos mx}{m} + C}$$

$u = mx \quad du = m\,dx$

$$\int \sin mx\,dx = \frac{1}{m}\int \sin u\,du = -\frac{\cos mx}{m} + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\boxed{\int \cos mx\,dx = \frac{\sin mx}{m} + C}$$

7.3 Integrating trigonometric functions

We start the section by reviewing important trigonometric identities as following:

$\sin^2 x + \sin^2 x = 1$	$\tan^2 x = \sec^2 x - 1$
$\sin 2x = 2\sin x\cos x$	$\cos 2x = \cos^2 x - \sin^2 x$
$\sin^2 x = \frac{1}{2}(1-\cos 2x)$	$\cos^2 x = \frac{1}{2}(1+\cos 2x)$
$\sin A \pm B = \sin A\cos B \pm \cos A\sin B$	$\cos A \pm B = \cos A\cos B \mp \sin A\sin B$

7.3.1 Integrating products of sines and cosines

If m and n are positive integers, the integral $\int \sin^m x\cos^n x\,dx$ can be evaluated by one of the following procedures, depending on whether m and n are even or odd.

$\int \sin^m x\cos^n x\,dx$	Procedure	relevant identity
m odd	set $\sin^m x = \sin^{m-1} x\sin x$	$\sin^2 x = 1-\cos^2 x$
n odd	set $\cos^n x = \cos^{n-1} x\cos x$	$\cos^2 x = 1-\sin^2 x$
m and n even	set $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ or set $\cos^2 x = \frac{1}{2}(1+\cos 2x)$	$\cos^2 x = \frac{1}{2}(1+\cos 2x)$ $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

Example 7.6 Evaluate $\int \sin^3 x\,dx$.

$(1-\cos^2 x)$

$$\int \sin^3 x\,dx = \int \underline{\sin^2 x}\cdot \overbrace{\sin x\,dx}^{-du}$$

$$= \int (1-\overbrace{\boxed{\cos^2 x}}^{u})\boxed{\sin x\,dx}$$

ให้ $u = \cos x$ $\quad \therefore du = -\sin x\,dx$

$$= -\int (1-u^2)\,du$$

$$= -\left[u - \frac{u^3}{3}\right] + C$$

$$= -\cos x + \frac{\cos^3 x}{3} + C \quad \#$$

Example 7.7 Evaluate $\int \cos^5 x\,dx$.

$$\int \cos^5 x\,dx = \int \cos^4 x \cdot \cos x\,dx$$

$$= \int (\cos^2 x)^2 \cos x\,dx$$

$$= \int (1-\underbrace{\sin^2 x}_{u})^2 \overbrace{\cos x\,dx}^{du} \quad —(*)$$

ให้ $u = \sin x$ $\quad \therefore du = \cos x\,dx$

$$(*) = \int (1-u^2)^2\,du$$

$$= \int (1-2u^2+u^4)\,du$$

$$= u - \frac{2}{3}u^3 + \frac{u^5}{5} + C$$

$$= (\sin x) - \frac{2}{3}(\sin x)^3 + \frac{(\sin x)^5}{5} + C \quad \#$$

Example 7.8 Evaluate $\int \sin^2 x \cos^3 x\,dx$.

$$\int \sin^2 x \cos^3 x\,dx = \int \sin^2 x \underbrace{\cos^2 x}_{1-\sin^2 x} \cdot \cos x\,dx$$

$$= \int \sin^2 x (1-\sin^2 x) \cos x\,dx$$

$$= \int (\underbrace{\sin^2 x}_{u} - \underbrace{\sin^4 x}_{u}) \overbrace{\boxed{\cos x\,dx}}^{du} \quad —(*)$$

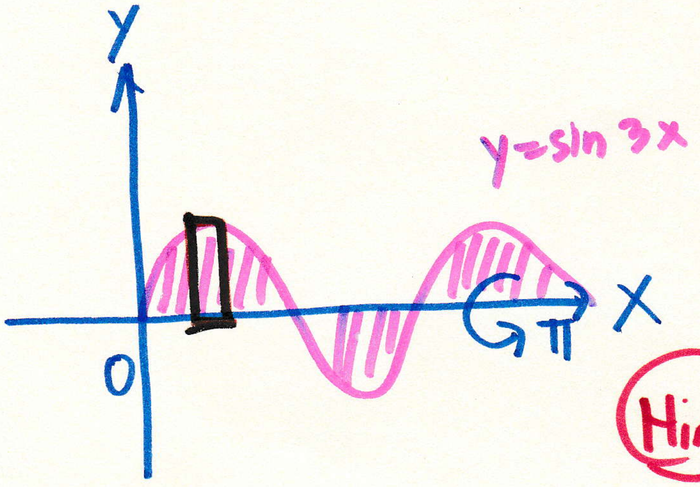
ให้ $u = \sin x$ $\quad du = \cos x\,dx$

$$(*) = \int (u^2 - u^4)\,du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C \quad \#$$

Example 7.9 Find the volume V of the solid that is obtained when the region under the curve $y = \sin 3x$ over the interval $[0, \pi]$ is revolved about the x-axis.



$$V = \int_0^{\pi} \pi (\sin 3x)^2 \, dx$$

$$= \pi \int_0^{\pi} \sin^2 3x \, dx$$

Hint : $\sin^2 3x = \frac{1}{2}[1 - \cos 6x]$

$$= \frac{\pi}{2} \int_0^{\pi} (1 - \cos 6x) \, dx$$

$$= \frac{\pi}{2} \left[x - \frac{\sin 6x}{6} \right]_0^{\pi}$$

$$= \frac{\pi}{2} \left[\left(\pi - \frac{\overset{0}{\sin 6\pi}}{6} \right) - (0 - 0) \right]$$

$$= \frac{\pi^2}{2} \quad \#$$