Exercise : Chapter 6

206171 (1/60) 頑張りましょう! Let's do our best

① $f(x) = x^3 - 3x^2 + 1$

(a) $f'(x) = 3x^2 - 6x$. Remark that $f'(x)$ exists for all $x \in \mathbb{R}$.

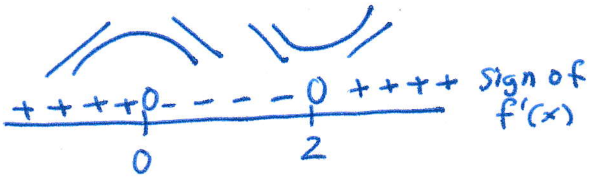
Find the critical numbers of $f(x)$.

Set $f'(x) = 0$: $3x^2 - 6x = 0$

$3x(x-2) = 0$

$\therefore x = 0, 2$

$\therefore$ The critical numbers are $x = 0, 2$.



$\therefore f'(x) > 0$ on $(-\infty, 0) \cup (2, \infty)$

$\Rightarrow f$ increases on $(-\infty, 0) \cup (2, \infty)$

$f'(x) < 0$ on $(0, 2)$

$\Rightarrow f$ decreases on $(0, 2)$

(b) f has local extrema at $x = 0, 2$.

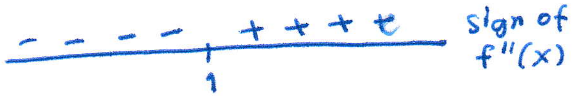
At $x = 0$, f has local maximum, and local maximum is $f(0) = 1$.
At $x = 2$, f has local minimum, and local minimum is $f(2) = -3$.

(c) $f''(x) = 6x - 6$. Remark that $f''(x)$ exists for all $x \in \mathbb{R}$.

Consider $f''(x) = 0 \Rightarrow$ $6x - 6 = 0$

$6x = 6$

$\therefore x = 1$



$\therefore f''(x) > 0$ on $(1, \infty)$

$\Rightarrow f$ is concave up on $(1, \infty)$

$f''(x) < 0$ on $(-\infty, 1)$

$\Rightarrow f$ is concave down on $(-\infty, 1)$

(d) Since $f''(x)$ change its sign at $x = 1$, the inflection point(s) is at $(1, f(1))$; i.e. inflection point is $(1, -1)$ #

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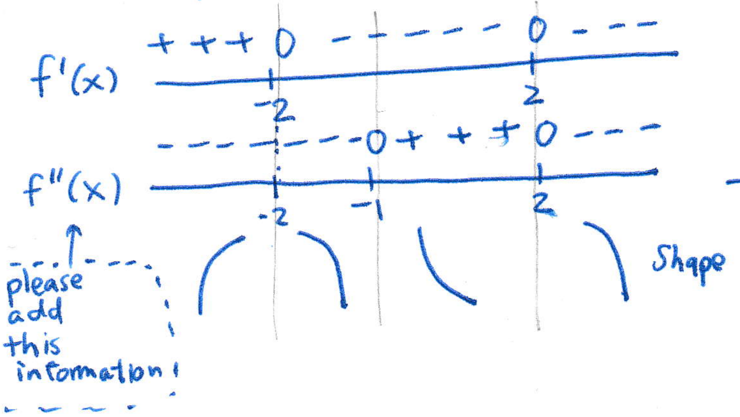
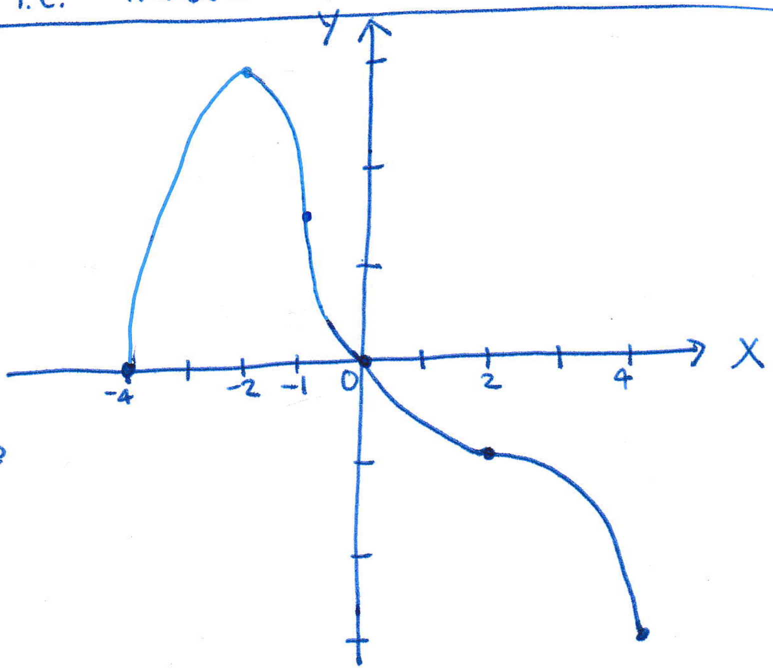
② (a)

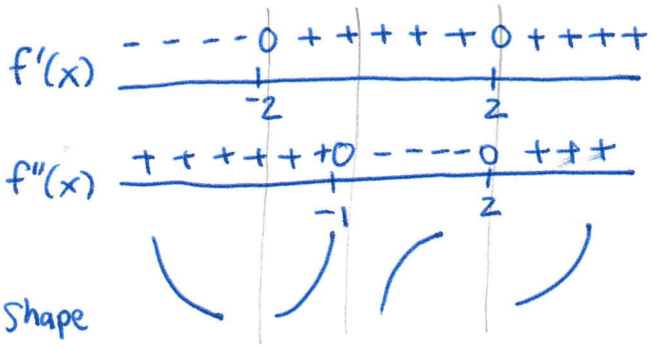
x	-4	-2	-1	0	2	4
f(x)	0	3	1.5	0	-1	-3

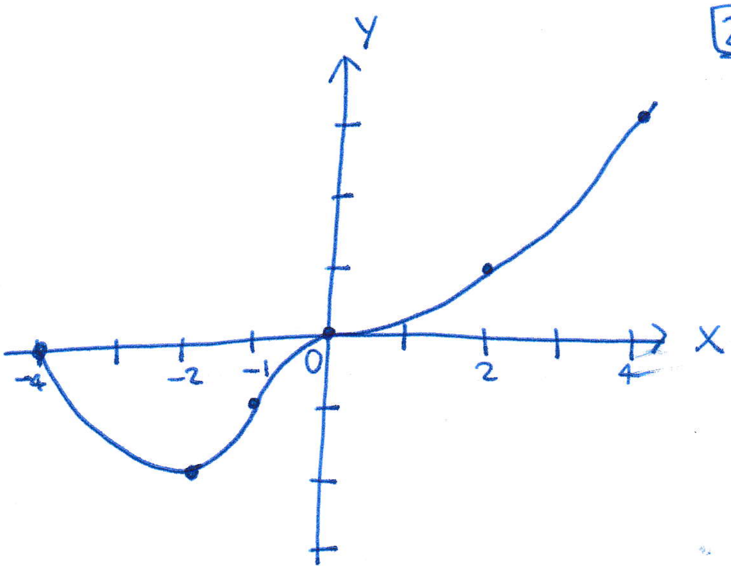




(b)

x	-4	-2	-1	0	2	4
$f(x)$	0	-2	-1	0	1	3





(c) $f(0) = 2$, $f(1) = 0$, $f(2) = -2$

$f'(0) = 0$, $f'(2) = 0$,

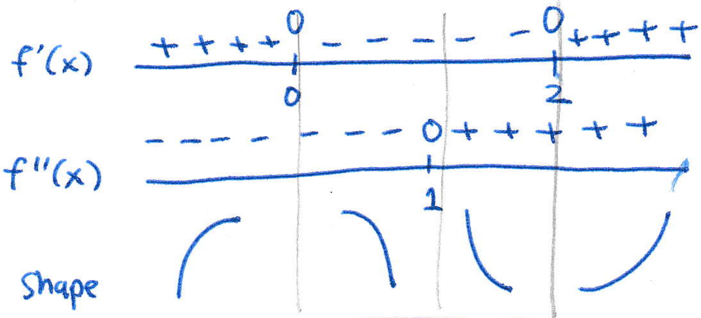
$f'(x) > 0$ on $(-\infty, 0)$ and $(2, \infty)$;

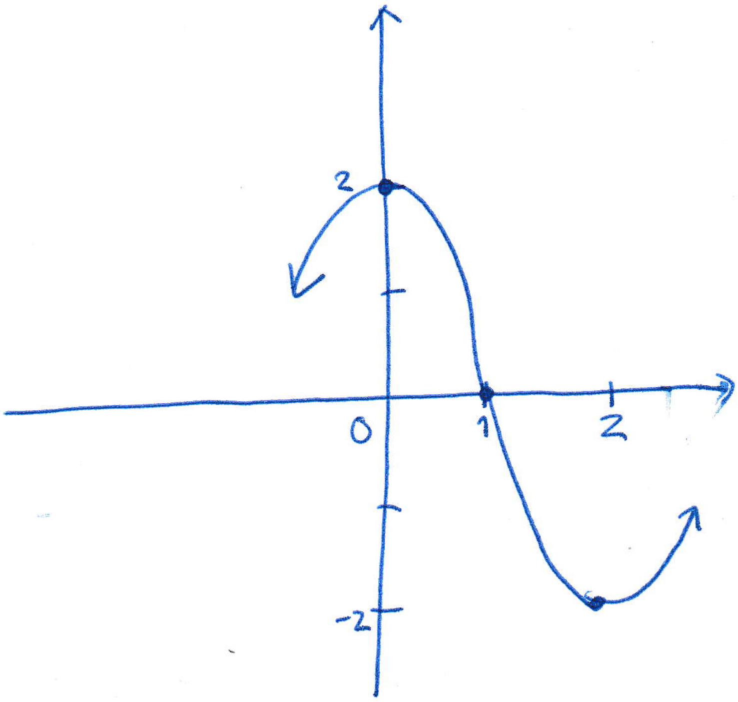
$f'(x) < 0$ on $(0, 2)$

$f''(x) > 0$ on $(1, \infty)$

$f''(x) < 0$ on. $(-\infty, 1)$

$f''(1) = 0$





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③ $P(x) = R(x) - C(x)$.

Since $R(x) = x \cdot p(x)$

$$= x\left(200 - \frac{x}{50}\right) = 200x - \frac{x^2}{50}$$

$$\therefore P(x) = \left(200x - \frac{x^2}{50}\right) - (60{,}000 + 60x)$$

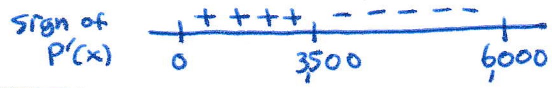
Hence, $\boxed{P(x) = -60{,}000 + 140x - \frac{x^2}{50}}$ s.t. $0 \leq x \leq 6{,}000$

Max P(x)?!? ① $P'(x) = 140 - \frac{2x}{50}$ (exists for all $x \in \mathbb{R}$)

Set $P'(x) = 0$; $140 = \frac{2x}{50}$ 25

$x = 3500$

② CHECK local extrema at $x = 3500$



RMKS $P(0) = -60{,}000$

(At boundaries) $P(6000) = -60{,}000 + 840{,}000 - \left(\frac{36000000}{50}\right) = 60{,}000$

③ P gives local maximum at $x = 3500$ such that local maximum is $P(3{,}500) = 185{,}000$ which is the absolute maximum on $[0, 6000]$

④
- At 3500 televisions will maximize the profit.
- The maximize profit is 185,000.
- The price that company needs to charge is $p(3500) = 200 - \frac{3500}{50} = 130$ #