ทบทวน Continuity.

$f: D \subseteq \mathbb{R}^n \to \mathbb{R}^n$ of $P \in \mathbb{R}^n$. f is continuous at $A \in \mathbb{R}^n$ iff.

1. $f(A)$ มีค่า
2. $\lim\limits_{P \to A} f(P)$ มีค่า
3. $\boxed{\lim\limits_{P \to A} f(P) = f(A).}$

f is continuous at $P = A \iff$

$$\forall \varepsilon > 0 \ \exists \delta > 0 \left[\|P - A\| < \delta \Rightarrow |f(P) - f(A)| < \varepsilon \right]$$

HW $f(x,y) = |x| \cdot (1+y)$. Prove that f is continuous at $(0,0)$.

Proof

(1) $f(0,0) = |0| \cdot (1+0) = 0$

(2) $\lim\limits_{(x,y) \to (0,0)} f(x,y) = 0$

Let $\varepsilon > 0$. choose $\delta = \min\{1, \frac{\varepsilon}{2}\}$

s.t. $|x| = \sqrt{x^2} \leq \sqrt{x^2+y^2} < \delta$

and $|y| = \sqrt{y^2} \leq \sqrt{x^2+y^2} < \delta$

ทด

$||x| \cdot (1+y)|$

$= \underbrace{|x|}_{<\delta} \cdot \underbrace{|1+y|}_{2}$

For $\delta \leq 1$; $|y| \leq 1$

$-1 \leq y \leq 1$

$-2 \leq 0 \leq y+1 \leq 2$

$|y+1| \leq 2$

$\Downarrow$

$-2 \leq y+1 \leq 2$

Consider that $|y+1| \leq 2$

Then $||x| \cdot (1+y)| = \overset{<\delta}{|x|} \cdot \underbrace{|1+y|}_{\leq 2}$

$\leq 2|x|$

$< 2\delta < \varepsilon$

$\therefore \lim\limits_{(x,y) \to (0,0)} |x| \cdot (1+y) = 0.$

(3) $f(0,0) = 0 = \lim\limits_{(x,y) \to (0,0)} f(x,y)$ $\therefore$ f cont

$\therefore$ f cont at $(0,0)$

Ex
$$f(x,y)=\begin{cases}\dfrac{x^2-y^2}{x^2+y^2} & ;(x,y)\neq(0,0)\\ 0 & ;(x,y)=(0,0)\end{cases}$$

Is f continuous at $(0,0)$?

Sol^n (1) $f(0,0)=0$

(2) $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$ ไหม.

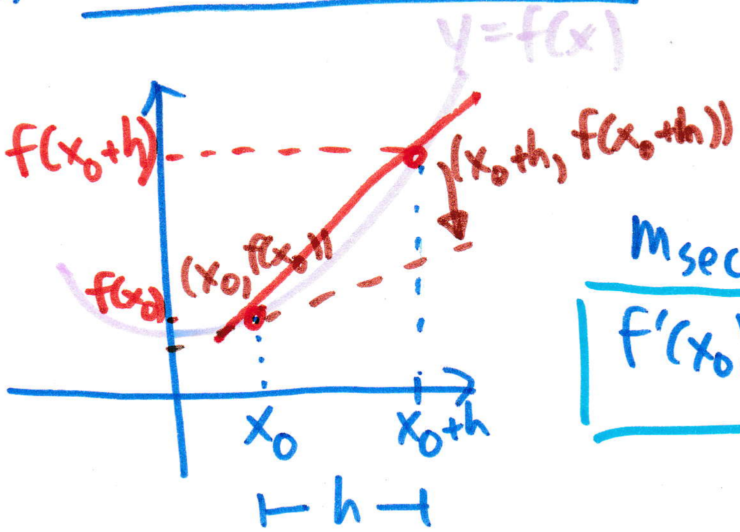
IL : $\lim\limits_{x\to 0}\left[\lim\limits_{y\to 0}\dfrac{x^2-y^2}{x^2+y^2}\right]=\lim\limits_{x\to 0} 1 = 1$

$\lim\limits_{y\to 0}\left[\lim\limits_{x\to 0}\dfrac{x^2-y^2}{x^2+y^2}\right]=\lim\limits_{y\to 0} -1 = -1$

$\therefore \lim\limits_{(x,y)\to(0,0)} f(x,y)$ DNE.

(3) f is not continuous at $(0,0)$.

(1.4) Partial Derivative



$$m_{sec} = \frac{f(x_0+h)-f(x_0)}{h}$$

$$\boxed{f'(x_0)=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}}$$

Def^n Let f be a function of two variables; $f = f(x,y)$.

"Partial Derivative" of f with respect to x is defined by

$$\frac{\partial f}{\partial x} = f_x(x,y) = D_x f(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

[If limit exists, $D_x f(x,y)$ exists].

"Partial derivative of f w.r.t. y is defined by.

$$\frac{\partial f}{\partial y} = f_y(x,y) = D_y f(x,y) = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

Ex Let $f(x,y) = 3x^2 - 2xy + y^2$. Evaluate $D_x f(x,y)$, $D_y f(x,y)$

Sol^n

$$\begin{aligned}
D_x f(x,y) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{[3(x+\Delta x)^2 - 2(x+\Delta x)y + y^2] - [3x^2 - 2xy + y^2]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{[3\{x^2 + 2x\Delta x + (\Delta x)^2\} - 2(x+\Delta x)y + y^2] - [3x^2 - 2xy + y^2]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{[\cancel{3x^2} + 6x\Delta x + 3(\Delta x)^2 - \cancel{2xy} - 2y\Delta x + \cancel{y^2} - \cancel{3x^2} + \cancel{2xy} - \cancel{y^2}]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{6x\cancel{\Delta x} + 3(\Delta x)^{\cancel{2}} - 2y\cancel{\Delta x}}{\cancel{\Delta x}} \\
&= \lim_{\Delta x \to 0} 6x + 3\Delta x - 2y \\
&= 6x - 2y
\end{aligned}$$

$\therefore D_x f(x,y) = 6x - 2y$.

HW $D_y f(x,y) = -2x + 2y$

At a particular point (x_0, y_0)

$$D_x f(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

$$D_y f(x_0, y_0) = \lim_{\Delta y \to 0} \frac{f(x_0, y_0+\Delta y) - f(x_0, y_0)}{\Delta y}$$

Let $\Delta x = x - x_0$, $\Delta y = y - y_0$ (เมื่อ $\Delta x \to 0$ ได้ว่า $x \to x_0$)

$$D_x f(x_0, y_0) = \lim_{x \to x_0} \frac{f(x, y_0) - f(x_0, y_0)}{x - x_0}$$

$$D_y f(x_0, y_0) = \lim_{y \to y_0} \frac{f(x_0, y) - f(x_0, y_0)}{y - y_0}.$$

Ex $f(x,y) = 3x^2 - 2xy + y^2$ จงหา $D_x f(3,-2)$ Hw