Quiz

A quiz for the class of February 23 (Fri), 2018.

Name....เฉลย~.... Student ID.................. No.........

A region $\mathcal{R}$ in the xy-plane is bounded by $x^2+y^2=a^2$, $x^2+y^2=b^2$, $x=0$, and $y=0$, where $0<a<b$.

1. Determine the region $\mathcal{R}'$ into which $\mathcal{R}$ is mapped under the transformation

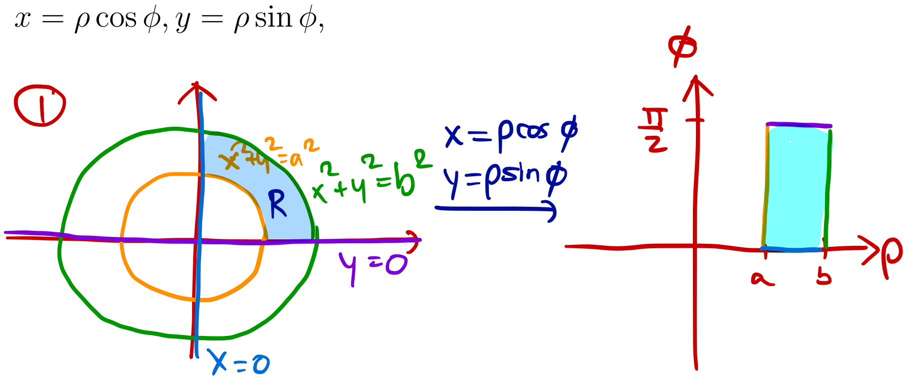
$$x=\rho\cos\phi, y=\rho\sin\phi,$$

where $\rho>0, 0\le\rho<2\pi$.

2. Compute $\dfrac{\partial(x,y)}{\partial(\rho,\phi)}$

3. Compute $\dfrac{\partial(\rho,\phi)}{\partial(x,y)}$

(1)



จากการแปลง $a^2 = x^2+y^2 = (\rho\cos\phi)^2+(\rho\sin\phi)^2 = \rho^2(\sin^2\phi+\cos^2\phi)$

$\therefore \rho = a$

ทำนองเดียวกัน $b^2 = x^2+y^2$ ได้ $\rho = b$

เมื่อ $x=0$ ได้ว่า $0=\rho\cos\phi$ จึงได้ว่า $\phi=\frac{\pi}{2}$

$y=0$ ได้ว่า $0=\rho\sin\phi$ จึงได้ว่า $\phi=0$

(2) $$\frac{\partial(x,y)}{\partial(\rho,\phi)} = J(\rho,\phi) = \begin{vmatrix} x_\rho & x_\phi \\ y_\rho & y_\phi \end{vmatrix} = \begin{vmatrix} \cos\phi & -\rho\sin\phi \\ \sin\phi & \rho\cos\phi \end{vmatrix} = \rho\cos^2\phi+\rho\sin^2\phi = \rho$$

(3) พิจารณา $x=\rho\cos\phi$, $y=\rho\sin\phi$ ดังนั้น $\tan\phi = \frac{y}{x} \Rightarrow \phi = \arctan\frac{y}{x}$

และ $x^2+y^2 = (\rho\cos\phi)^2+(\rho\sin\phi)^2 = \rho^2$ ได้ว่า $\rho=\sqrt{x^2+y^2}$ (เพราะว่า $\rho>0$)

$$\therefore \frac{\partial(\rho,\phi)}{\partial(x,y)} = J(x,y) = \begin{vmatrix} \rho_x & \rho_y \\ \phi_x & \phi_y \end{vmatrix} = \begin{vmatrix} \frac{1}{\cancel{2}\sqrt{x^2+y^2}}(\cancel{2}x) & \frac{1}{\cancel{2}\sqrt{x^2+y^2}}(\cancel{2}y) \\ \left(\frac{1}{1+(\frac{y}{x})^2}\right)\left(-\frac{y}{x^2}\right) & \left(\frac{1}{1+(\frac{y}{x})^2}\right)\left(\frac{1}{x}\right) \end{vmatrix}$$

$$= \begin{vmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{vmatrix} = \left(\frac{1}{\sqrt{x^2+y^2}}\right)\left(\frac{1}{\cancel{x^2+y^2}}\right)\cancel{(x^2+y^2)}$$

$$= \frac{1}{\sqrt{x^2+y^2}} \quad \left(\text{นั่นคือ} = \frac{1}{\rho}\right).$$

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⊛ จะเห็นว่า $\rho = J(\rho,\phi) = \dfrac{1}{J(x,y)} = \dfrac{1}{\frac{1}{\rho}}$ นั่นเอง!