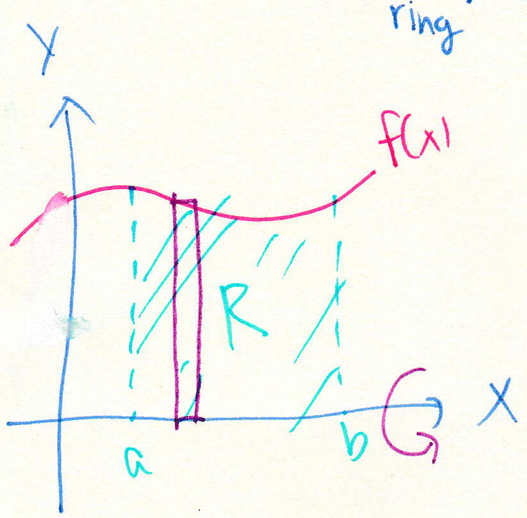
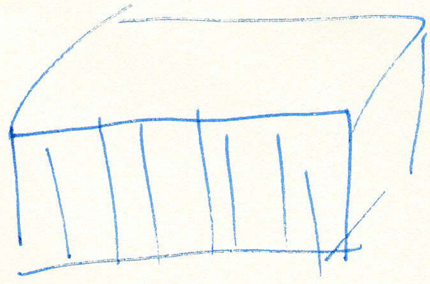
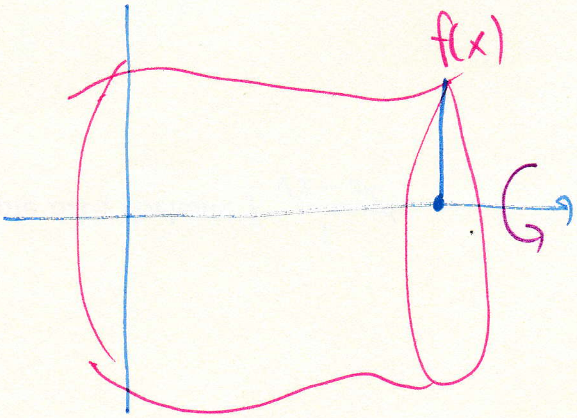
ปริมาตรทรงตันจากการหมุนแบบ Disk → บริเวณที่หมุนอยู่ติดแกนหมุน

Washer/ring → บริเวณที่หมุนไม่อยู่ติดกับแกนหมุน.



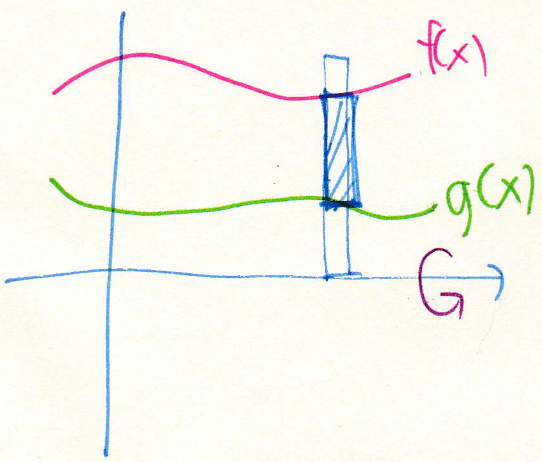




Disk

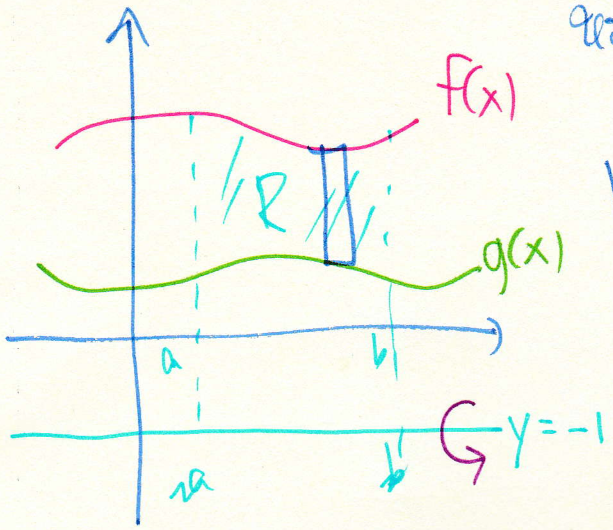
$$V=\int_a^b \pi\left[f(x)\right]^2 dx$$

Washer



$\underbrace{f(x)-g(x)}$ ก็จริง แต่เวลาหาปริมาตรต้องไปยกกำลังสอง

$$\therefore V=\pi\int_a^b \left[(f(x))^2-(g(x))^2\right]dx$$



หมุนรอบ $y = -1$

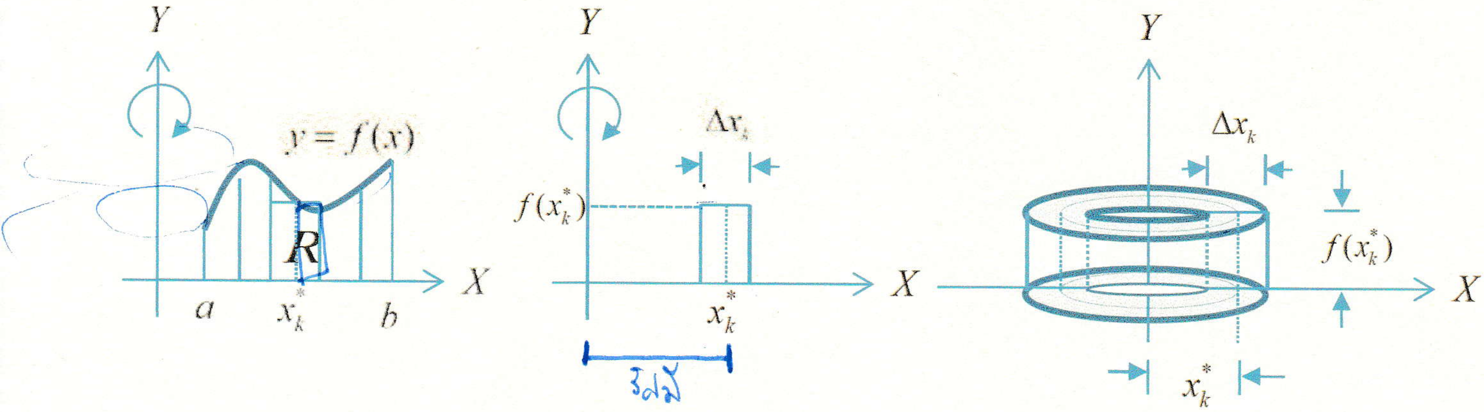
$$V = \pi \int_a^b \left[[f(x) - (-1)]^2 - [g(x) - (-1)]^2 \right] dx$$

เปลือกทรงกระบอก

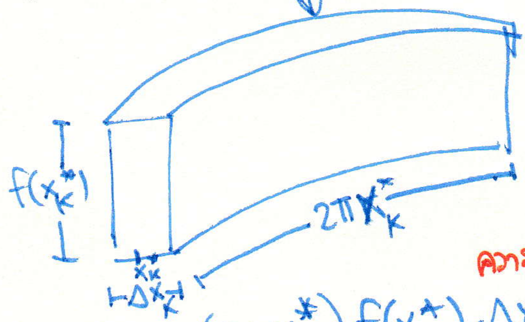
6.3 Volumes by Cylindrical Shells * สร้างแถบให้ขนานกับแกนหมุน

Theorem 6.5 (**Volume by cylindrical shells about the Y-axis**) Let f be continuous and nonnegative on [a, b] and let R be the region that is bounded above by $y = f(x)$, below by the X-axis, and on the sides by the lines $x = a$ and $x = b$. Then the volume V of the solid of revolution that is generated by revolving the region R about the Y-axis is given by

$$V = \int_a^b 2\pi x f(x)dx. \qquad (6.9)$$



$$V = \lim_{max\Delta x_k \to 0} \sum_{k=1}^{n} 2\pi x_k^* f(x_k^*)\Delta x_k = \int_a^b 2\pi x f(x)dx.$$



ความกว้าง

$$\therefore V_k = \underbrace{(2\pi x_k^*)}_{\text{ความยาว}} \underbrace{f(x_k^*)}_{\text{ความสูง}} \cdot \Delta x_k$$

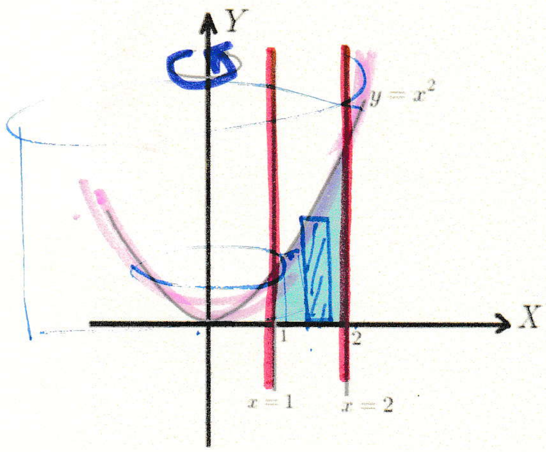
$$V_{\text{รวม}} = \lim_{max \Delta x_k \to 0} \sum_{k=1}^{n} 2\pi x_k^* f(x_k^*) \Delta x_k$$

รัศมี

$$\boxed{V_{\text{รวม}} = \int_a^b \underbrace{2\pi x}_{\text{ความยาวแท่ง}} \underbrace{f(x)}_{\text{ความสูง}} \, dx}$$

ความกว้างแท่ง.

Example 6.11 Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^2$, $x = 1$, $x = 2$ and the X-axis is revolved about the Y-axis.



$$V = \int_1^2 2\pi x\,[x^2 - 0]\,dx$$

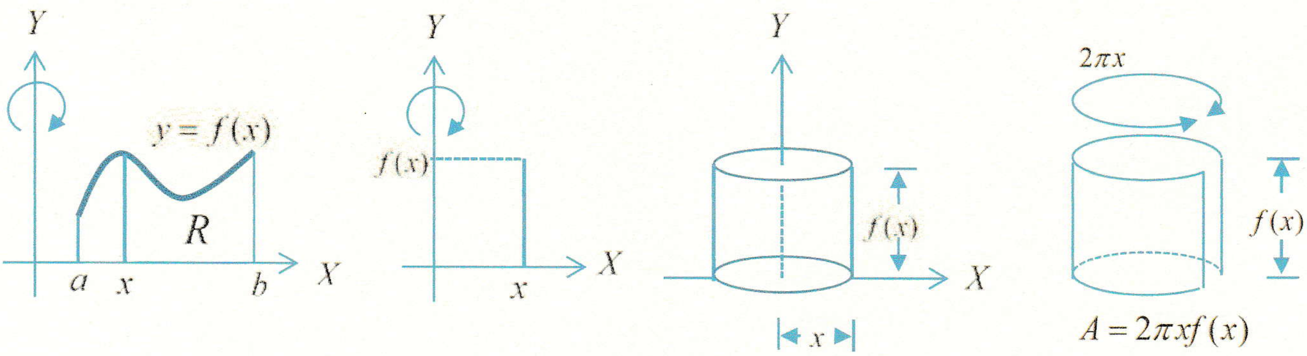
$$= 2\pi \int_1^2 x^3\,dx$$

$$= 2\pi \left[\frac{x^4}{4}\right]_1^2$$

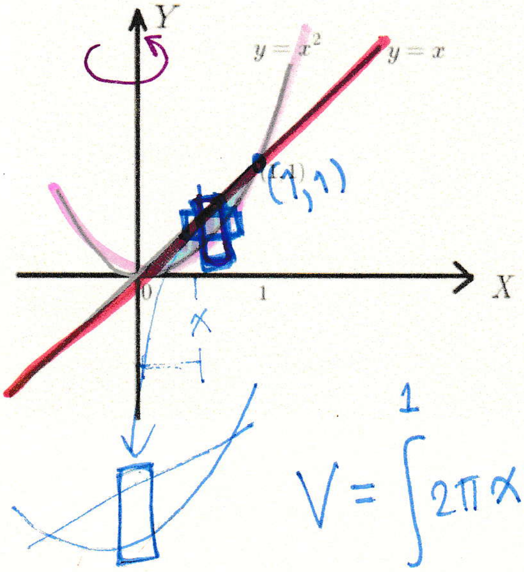
$$= 2\pi \left[\frac{16}{4} - \frac{1}{4}\right]$$

$$= \frac{15\pi}{2}$$

An informal viewpoint about cylindrical shells: The volume V of a solid of revolution that is generated by revolving a region R about an axis can be obtained by integrating the area of the surface generated by an arbitrary cross section of R taken parallel to the axis of revolution.



Example 6.12 Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the Y-axis.



จุดตัดระหว่าง $y=x^2$ กับ $y=x$

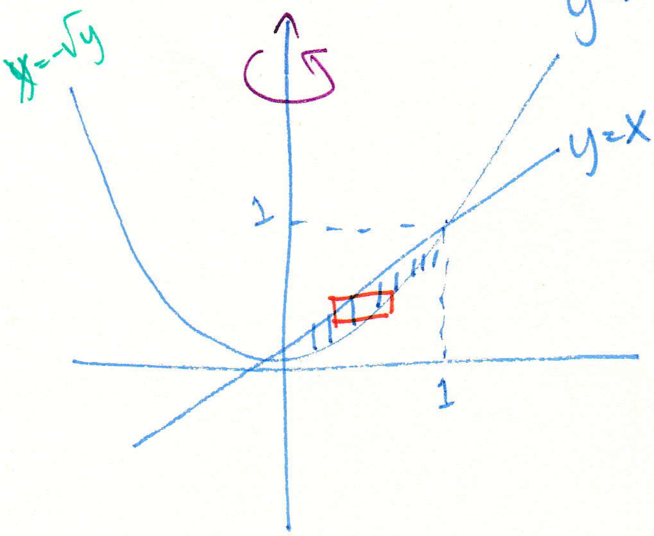
$$x^2 = x$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0, 1$$

ที่ $x=1$ ได้ว่า $y=1$

$$V = \int_0^1 2\pi x(x - x^2)\, dx$$
$$= 2\pi \int_0^1 (x^2 - x^3)\, dx$$
$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 2\pi\left[\frac{1}{3} - \frac{1}{4}\right] = 2\pi\left[\frac{4-3}{12}\right]$$
$$= \frac{\pi}{6} \quad \#$$

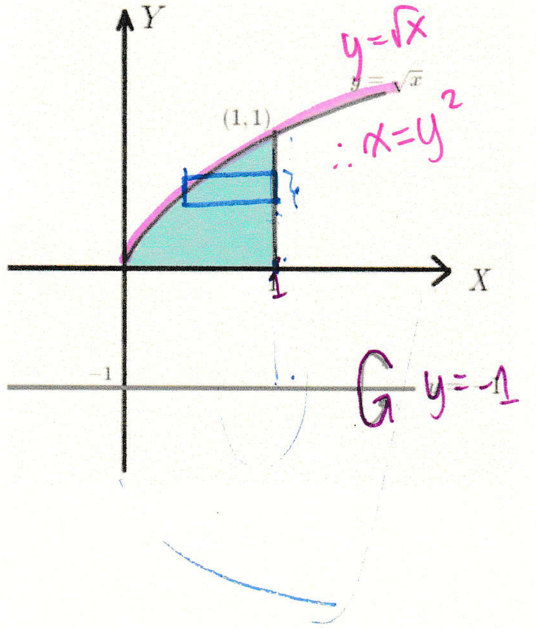
หมุนแบบ washer รอบแกน y

$y = x^2 \Rightarrow x = \sqrt{y}$



$$V = \int_0^1 \pi\left[(\sqrt{y})^2 - (y)^2\right] dy$$
$$= \pi \int_0^1 [y - y^2]\, dy$$
$$= \pi\left[\frac{y^2}{2} - \frac{y^3}{3}\right]_0^1$$
$$= \pi\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{\pi}{6} \quad \#$$

Example 6.13 Use cylindrical shells to find the volume of the solid generated when the region R under $y = \sqrt{x}$ over the interval $[0, 1]$ is revolved about the line $y = -1$.



$$V = \int_0^1 2\pi \underbrace{(y-(-1))}_{\text{รัศมี}} \underbrace{[1-y^2]}_{\text{ความสูง}} \, dy$$

$$= 2\pi \int_0^1 (y+1)(1-y^2)\, dy$$

$$= 2\pi \int_0^1 [(y+1)-(y+1)y^2]\, dy$$

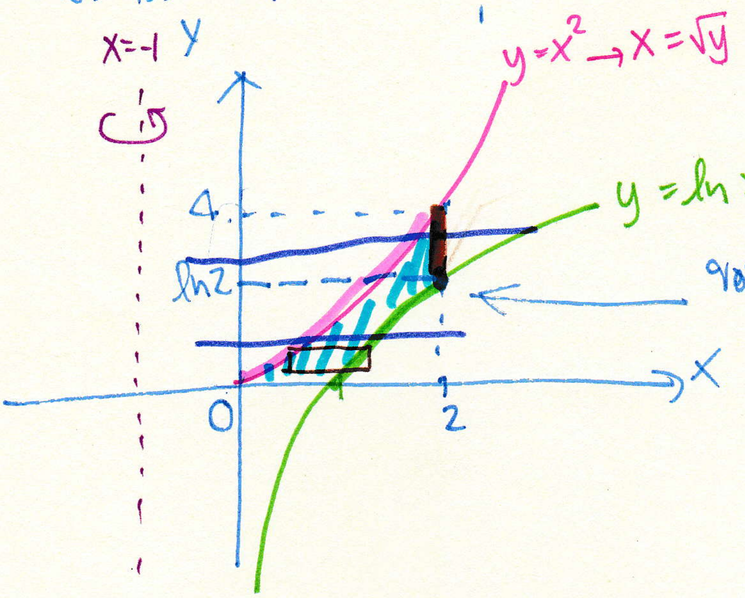
$$= 2\pi \int_0^1 [y+1-y^3-y^2]\, dy$$

$$= 2\pi \left[\frac{y^2}{2} + y - \frac{y^4}{4} - \frac{y^3}{3}\right]_0^1$$

$$= 2\pi \left[\frac{1}{2} + 1 - \frac{1}{4} - \frac{1}{3}\right]$$

$$= \cancel{2}\pi \left[\frac{6+12-3-4}{\cancel{12}\,6}\right] = \frac{11\pi}{6} \quad \#$$

Ex จงหาปริมาตรของทรงตันที่เกิดจากการหมุน บริเวณต่อไปนี้ รอบแกน $x = -1$



ขอบเขต คือจุดที่ $y = \ln x$ ตัดกับ $x = 2$

$\therefore y = \ln 2$

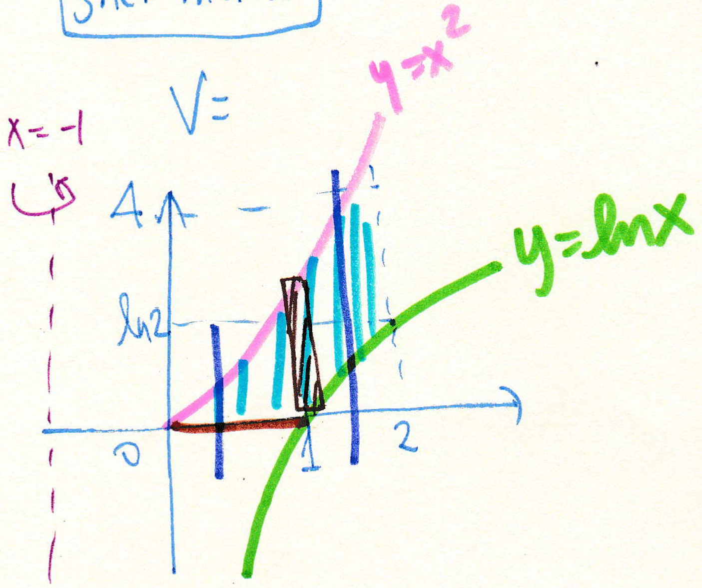
(washer)

$\boxed{\text{Disk Method}}$ อินทิเกรตเทียบ y

$$V = \int_0^{\ln 2} \pi\left[\left(e^y-(-1)\right)^2 - \left(\sqrt{y}-(-1)\right)^2\right] dy + \int_{\ln 2}^{4} \pi\left[\left[2-(-1)\right]^2 - \left[\sqrt{y}-(-1)\right]^2\right] dy$$

$\boxed{\text{Shell method}}$

$V =$



$y = \ln x \rightarrow e^y = x$

$\log_{\text{ฐาน}} x = \text{ริกเอ}$

$x = \text{ฐาน}^{\text{ริกเอ}}$

$$V = \int_0^1 2\pi\left(x-(-1)\right)\left[x^2 - 0\right] dx + \int_1^2 2\pi\left(x-(-1)\right)\left[x^2 - \ln x\right] dx.$$

7

เทคนิคการอินทิเกรต

Techniques of Integration

7.1 An overview of integration methods

A review of familiar integration formulas

1. $\int du = u + C$	2. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
3. $\int \frac{1}{u}\, du = \ln\lvert u\rvert + C$	4. $\int a^u\, du = \frac{a^u}{\ln a} + C,\ a > 0, a \neq 1$
5. $\int e^u\, du = e^u + C$	
6. $\int \sin u\, du = -\cos u + C$	7. $\int \cos u\, du = \sin u + C$
8. $\int \sec^2 u\, du = \tan u + C$	9. $\int \csc^2 u\, du = -\cot u + C$
10. $\int \sec u \tan u\, du = \sec u + C$	11. $\int \csc u \cot u\, du = -\csc u + C$
12. $\int \tan u\, du = ln\lvert \sec u\rvert + C$	13. $\int \cot u\, du = ln\lvert \sin u\rvert + C$
14. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin(\frac{u}{a}) + C$	15. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan(\frac{u}{a}) + C$

7.2 Integration by parts การอินทิเกรตแบบแยกส่วน.

7.2.1 The product rule and integration by parts

Let $G(x)$ be any antiderivative of $g(x)$; $G'(x) = g(x)$

$$\frac{d}{dx}[f(x)G(x)] = f(x)G'(x) + f'(x)G(x) = f(x)g(x) + f'(x)G(x)$$

$$\int [f(x)g(x) + f'(x)G(x)]\ dx = f(x)G(x)$$

$$\int f(x)g(x)\ dx = f(x)G(x) - \int f'(x)G(x)\ dx$$

Let $u = f(x), du = f'(x)dx$ and $v = G(x), dv = g(x)\ dx$

$$\int udv = uv - \int vdu$$

ห้ามทำ

$$\int x \cos x\, dx = \cancel{x\int \cos x\, dx + \cos x \int x\, dx}$$ ห้ามทำ

$$= \cancel{\int x\, dx \cdot \int \cos x\, dx}$$

สมมติว่ามีฟังก์ชันคูณกันอยู่ 2 ตัว $u, v \longrightarrow u \cdot v$

$$d(uv) = u\,dv + v\,du$$

$$uv = \int d(uv) = \int u\,dv + \int v\,du.$$

$$\therefore \boxed{\int u\,dv = uv - \int v\,du}$$

Ex $\int x \cos x\, dx$ เลือกฟังก์ชันให้ตัวนึงเป็น u
เลือกอีกตัวเป็น v { เอาไปหาค่าอะไรก่อนดี.
?

Example 7.1 Evaluate $\int \overset{u}{x} \overset{dv}{\sin x\, dx}$.

By parts เลือก u , เลือก dv

เลือก $u = x \rightarrow du = dx$ ✓

$dv = \sin x\, dx \rightarrow v = \int dv$

$= \int \sin x\, dx$

$= -\cos x$

$$\circledast \int u\,dv = uv - \int v\,du$$

$$\therefore \int x \sin x\, dx = \overset{u}{x}\overset{v}{(-\cos x)} - \int \overset{v}{(-\cos x)}\,\overset{du}{dx}$$

$$= -x\cos x + \int \cos x\, dx$$

$$= -x\cos x + \sin x + C. \ \#$$

$u = \sin x \qquad dv = x\, dx$

$du = \cos x\, dx \qquad v = \frac{x^2}{2}$

$$\therefore \int x \sin x\, dx = \frac{x^2}{2}\sin x - \int \frac{x^2}{2}\cos x\, dx$$

$u = 1 \qquad dv = x \sin x\, dx$

$du = 0\, dx \qquad v = \int x \sin x\, dx$

$u = x\sin x \qquad dv = dx$

$\therefore du = (x\cos x + \sin x)dx \ ; \quad v = x$

$$\therefore \int x\sin x\, dx = x^2 \sin x - \int x\,(x\cos x + \sin x)\,dx$$