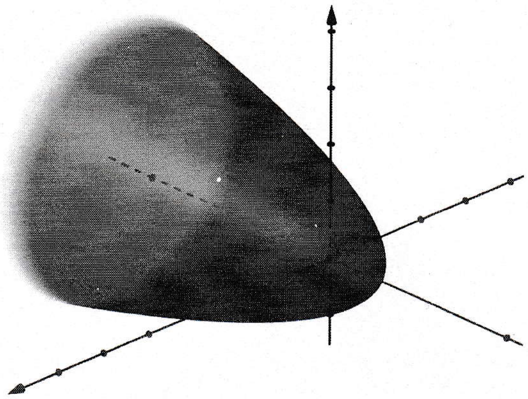
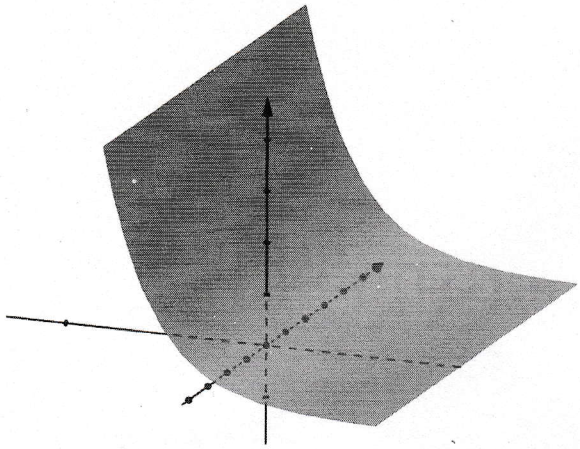
Surface parameterization

$x^2+z^2=1-y$, $-4\le y\le 1$ $\qquad$ $z=e^y$, $-1\le x\le 1$ $\qquad$ unit normal

vector





จงหาการแทนแบบอิงตัวแปรเสริมพร้อมทั้งบอกขอบเขตของตัวแปรเสริมของพื้นผิวต่อไปนี้

พื้นผิวทรงกระบอกรี $\frac{y^2}{9}+z^2=1,\quad -1\le x\le 3$

พื้นผิว $x+2y=z^3,\quad -1\le x\le 1,\ 0\le y\le 1$

พื้นผิว $y=9-x^2-z^2,\quad 0\le y\le 9$

- Line Integral $\int_C \vec{F} \cdot d\vec{r}$

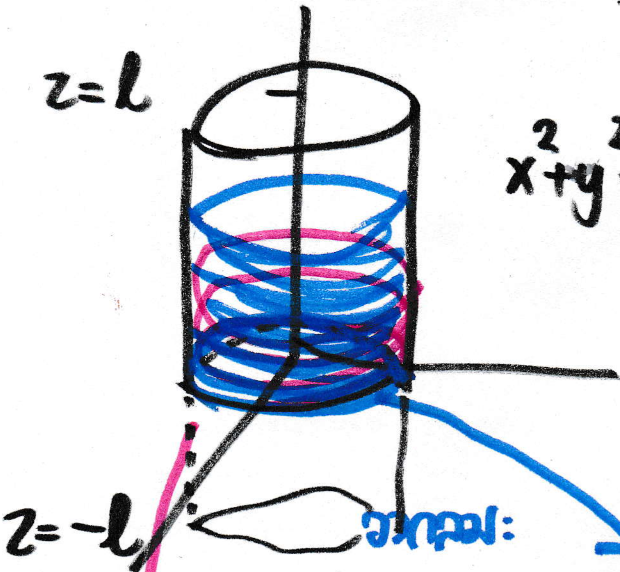
① Parametrization

$\vec{r}(t)$; $\underline{a \le t \le b}$

vector function $\vec{F}$ → ② $\vec{F}(t)$

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(t) \cdot \vec{r}\,'(t)\, dt$$

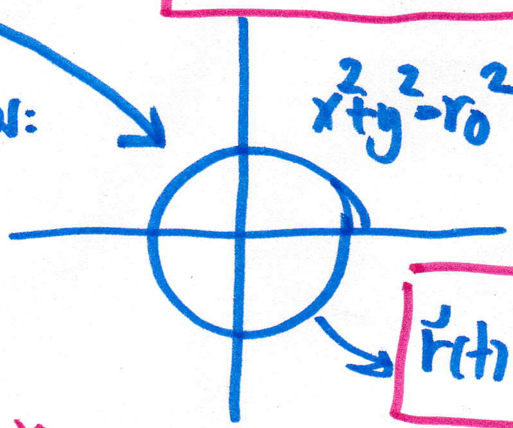
Surface $\underline{\text{Parametrization}}$: $\vec{r}(u,v)$



$-l \le v \le l$ — v ของ z

$x^2+y^2=r_0^2$, $-l \le z \le l$

$$\boxed{\vec{r}(u,v) = r_0 \cos u\, \vec{i} + r_0 \sin u\, \vec{j} + v\vec{k}}$$



$x^2+y^2=r_0^2$

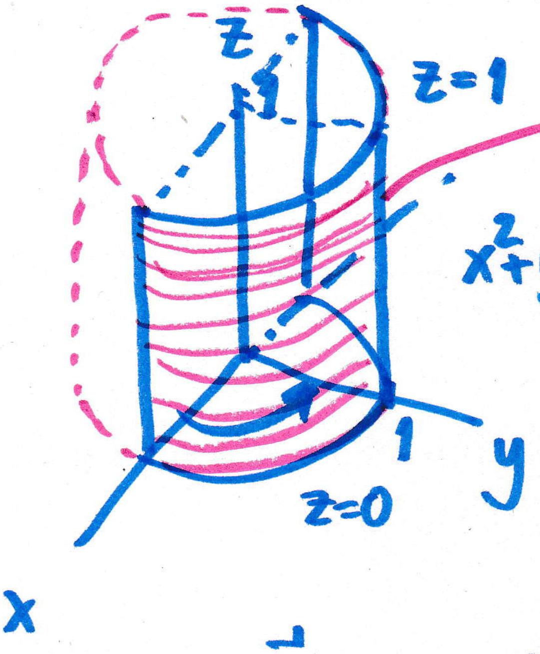
u: มุม $0 \le u \le 2\pi$

$$\boxed{\vec{r}(t) = r_0 \cos t\, \vec{i} + r_0 \sin t\, \vec{j}}$$

Helix: $\vec{r}(t) =$
$r_0 \cos t\, \vec{i} + r_0 \sin t\, \vec{j} + ct\,\vec{k}$

$0 \le t \le 2\pi$

ตัวแปรใหม่



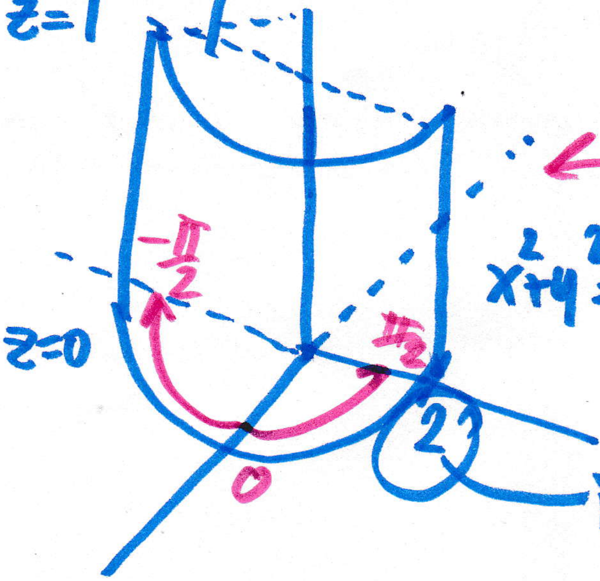
$0 \leq u \leq \pi$

$x^2+y^2=1 \;;\; 0 \leq z \leq 1$

$\vec{r}(u,v) = \cos u\,\vec{i} + \sin u\,\vec{j} + v\vec{k}$

$0 \leq u \leq \frac{\pi}{2}$

$0 \leq v \leq 1$

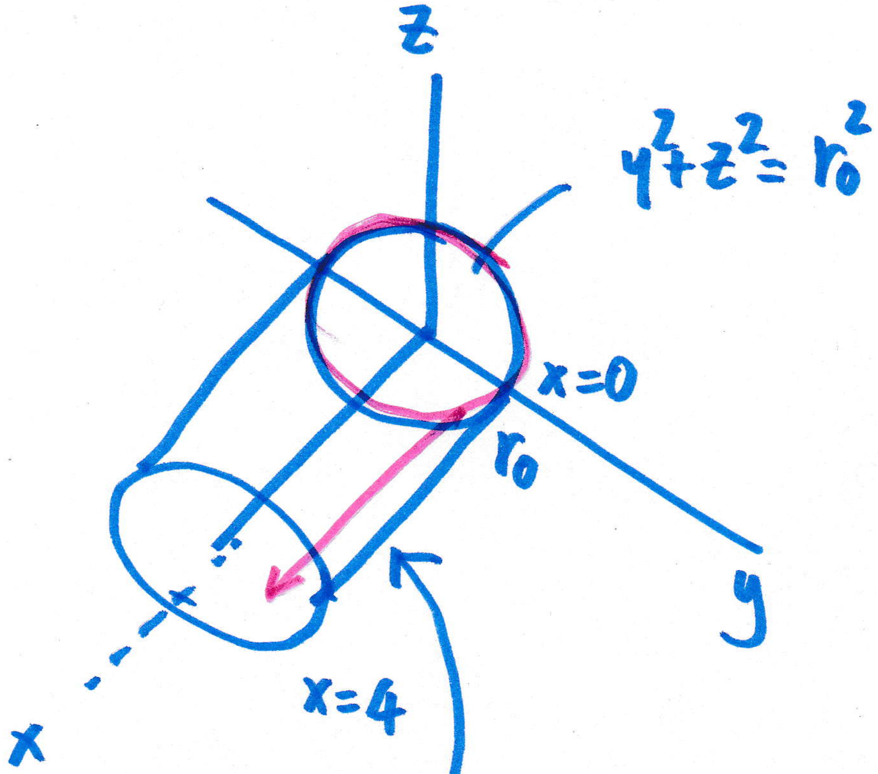


$x^2+y^2=4$

$-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$

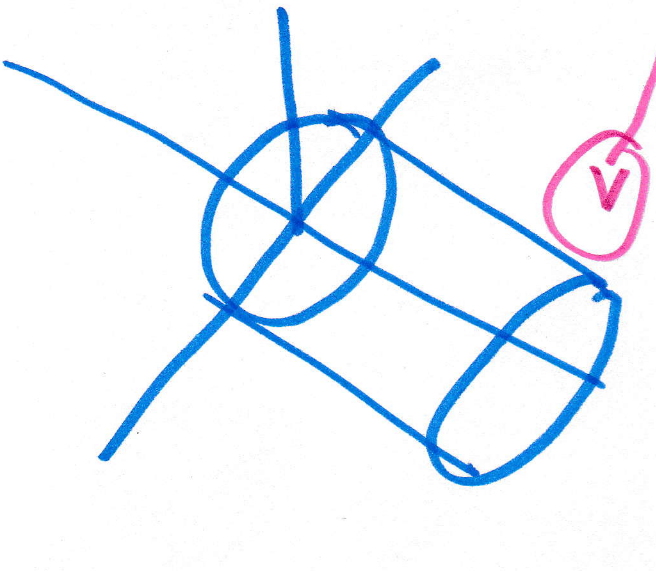
$0 \leq v \leq 1$

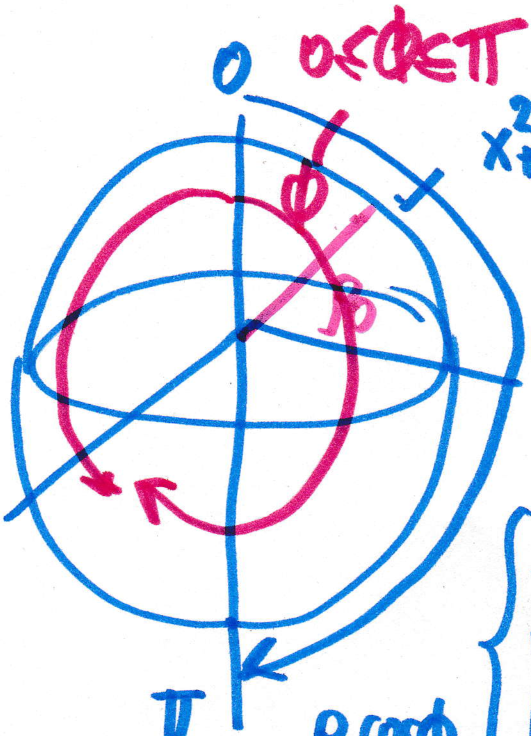
$\vec{r}(u,v) = 2\cos u\,\vec{i} + 2\sin u\,\vec{j} + v\vec{k}$



$$\vec{r}(u,v) = \underline{\underline{v}}\,\vec{i} + r_0 \cos u\,\vec{j} + r_0 \sin u\,\vec{k}$$

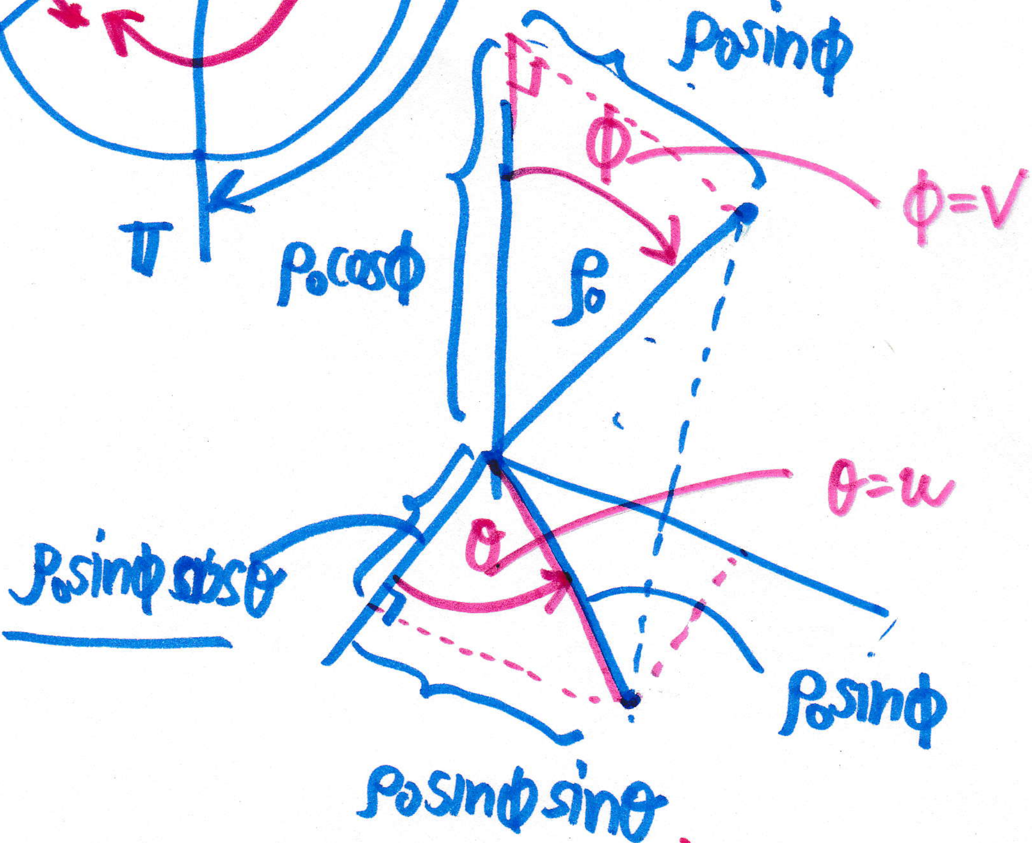
$0 \leq v \leq 4$; $0 \leq u \leq 2\pi$





$$x^2+y^2+z^2=\underline{\rho_0^2}$$

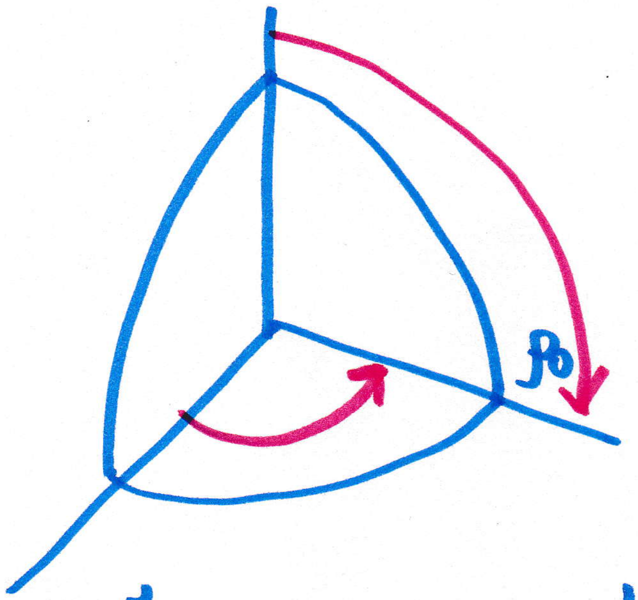
$$\vec{r}(u,v)=x(u,v)\vec{i}+y(u,v)\vec{j}+z(u,v)\vec{k}$$



$$\vec{r}(u,v)=\overset{x}{\rho_0\sin v\cos u\,\vec{i}}+\overset{y}{\rho_0\sin v\sin u\,\vec{j}}+\overset{z}{\rho_0\cos v\,\vec{k}}$$

$0\le u\le 2\pi \qquad 0\le v\le \pi$

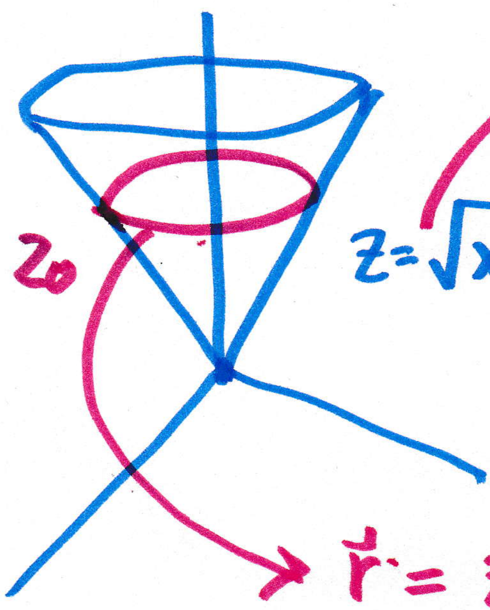
check $x^2+y^2+z^2=\rho_0^2$



$\vec{r}(u,v) = \rho_0 \sin v \cos u\, \vec{i} + \rho_0 \sin v \sin u\, \vec{j} + \rho_0 \cos v\, \vec{k}$

$$\boxed{0 \le u \le \frac{\pi}{2} \qquad 0 \le v \le \frac{\pi}{2}}$$ ***

กรวยกลม



$x^2+y^2=z^2$

$z=\sqrt{x^2+y^2}$

$\vec{r} = z_0\cos u\,\vec{i} + z_0\sin u\,\vec{j} + z_0\vec{k}$

$\vec{r}(u,v) = \underbrace{u\cos v}_{x}\,\vec{i} + \underbrace{u\sin v}_{y}\,\vec{j} + \underbrace{u}_{z}\vec{k}$ ตรงกับ z

$0 \le u \le H, \quad 0 \le v \le 2\pi$

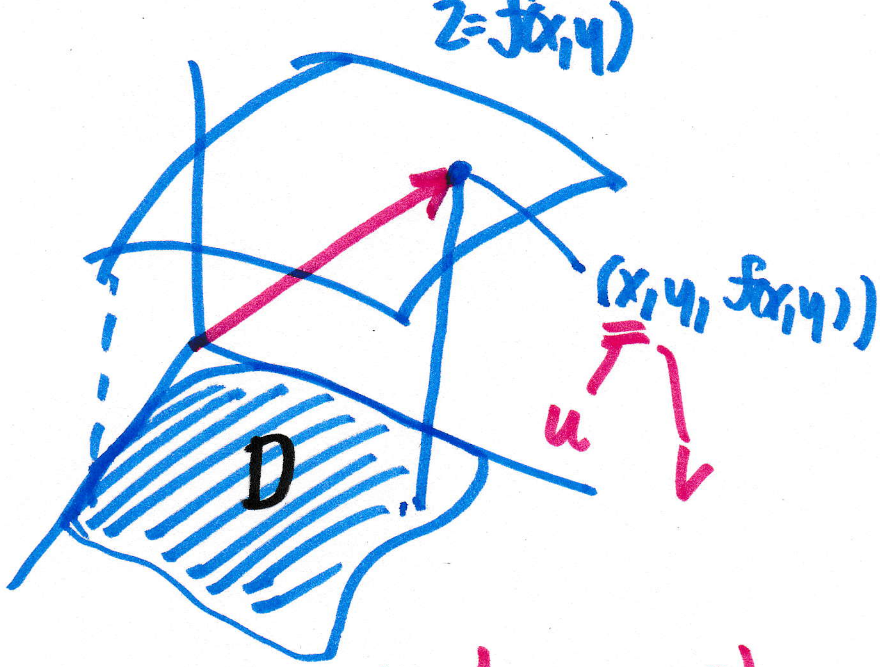
$x = u\cos v; \quad y = u\sin v; \quad z = u$

Check: $\sqrt{x^2+y^2} = \sqrt{u^2\cos^2 v + u^2\sin^2 v}$

$= \sqrt{u^2(\cos^2 v + \sin^2 v)} = \sqrt{u^2} = u$

z

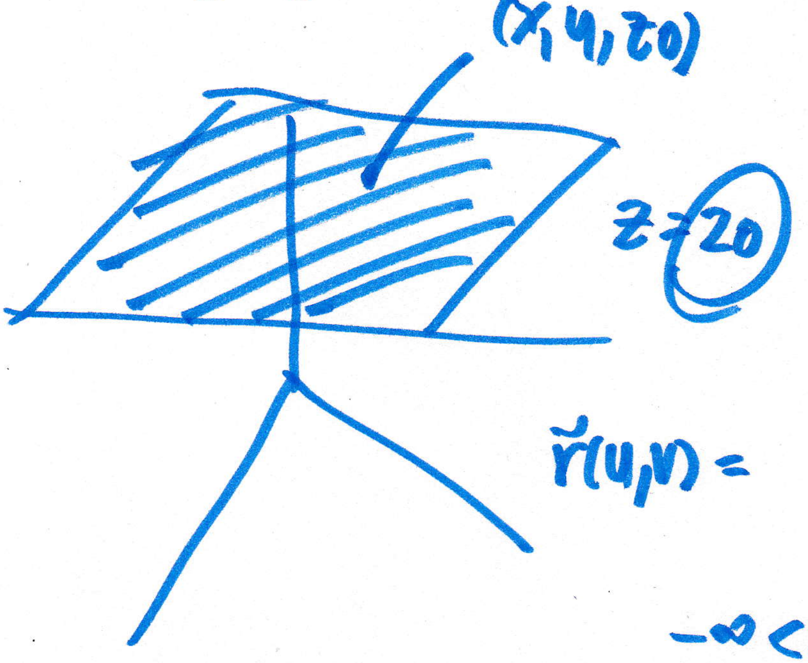
Why? $\vec{r}(u,v)$



$\vec{r}(u,v) = u\hat{i} + v\hat{j} + f(u,v)\vec{k}$

(u,v) อยู่ใน D

ระนาบ $z=z_0$

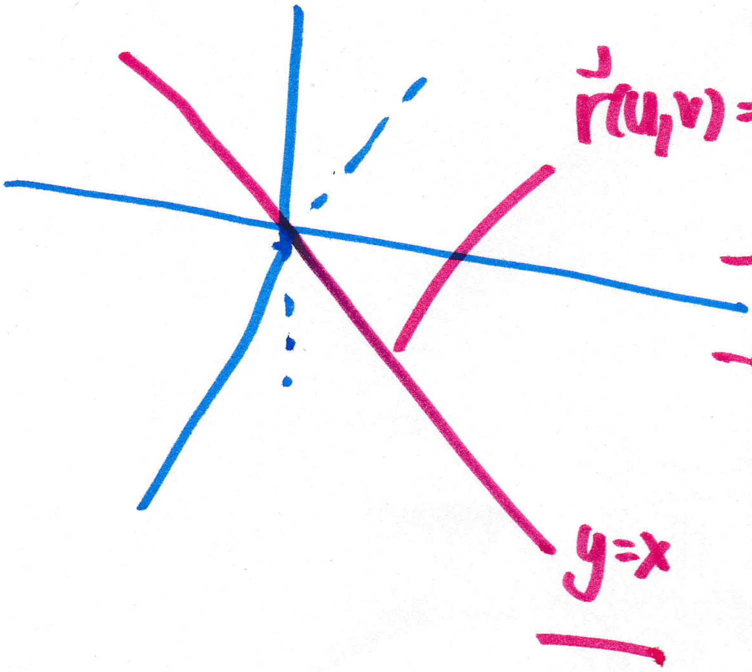


$$\vec{r}(u,v) = u\vec{i} + v\vec{j} + z_0\vec{k}$$

$-\infty < u < \infty$

$-\infty < v < \infty$

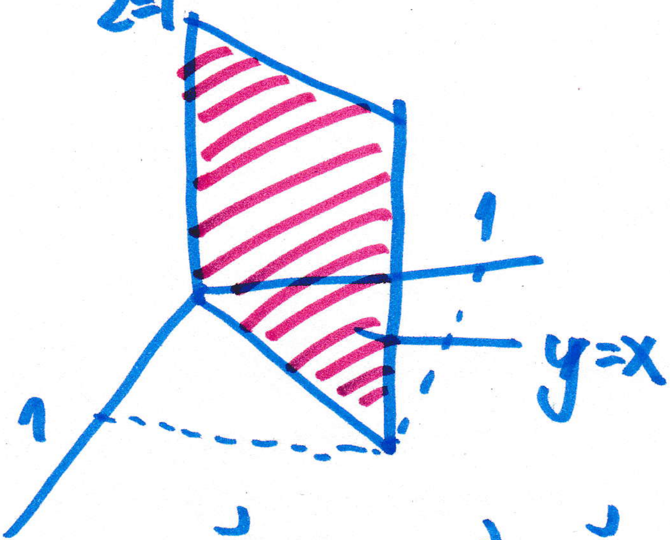
ระนาบ $y=x$



$$\vec{r}(u,v) = u\vec{i} + u\vec{j} + v\vec{k}$$

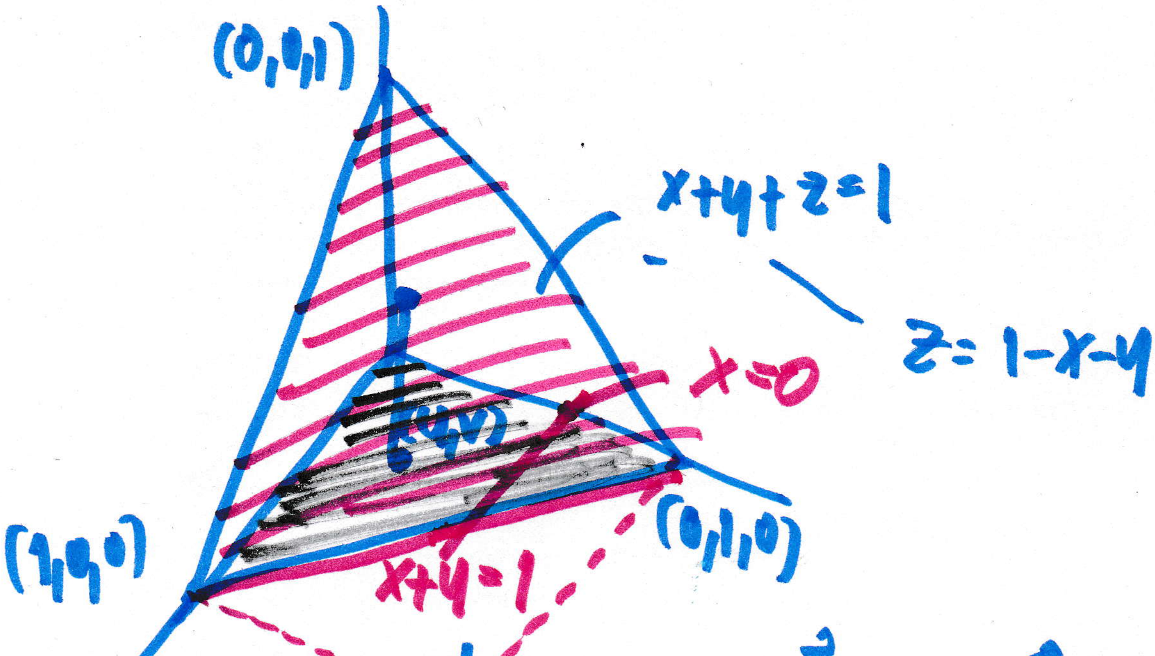
$-\infty < u < \infty$

$-\infty < v < \infty$



$$\vec{r}(u,v) = u\vec{i} + u\vec{j} + v\vec{k}$$

$0 \le u \le 1 \qquad 0 \le v \le 1$

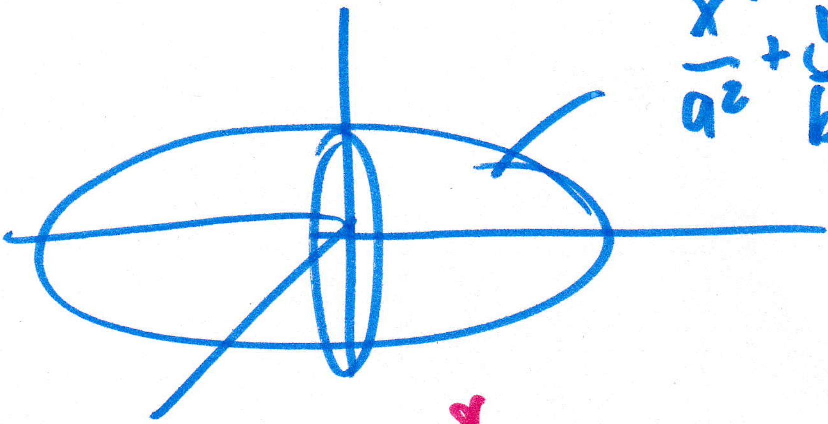


$$\vec{r}(u,v) = \underline{\ u\ }\,\vec{i} + \underline{\ v\ }\,\vec{j} + \underline{(1-u-v)}\,\vec{k}$$

กำหนด

~~$0 \le u \le 1 \quad 0 \le v \le 1$~~ X

$0 \le u \le 1-v; \quad 0 \le v \le 1$



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

เทียบ ทรงกลม: $x^2+y^2+z^2=\rho_0^2$

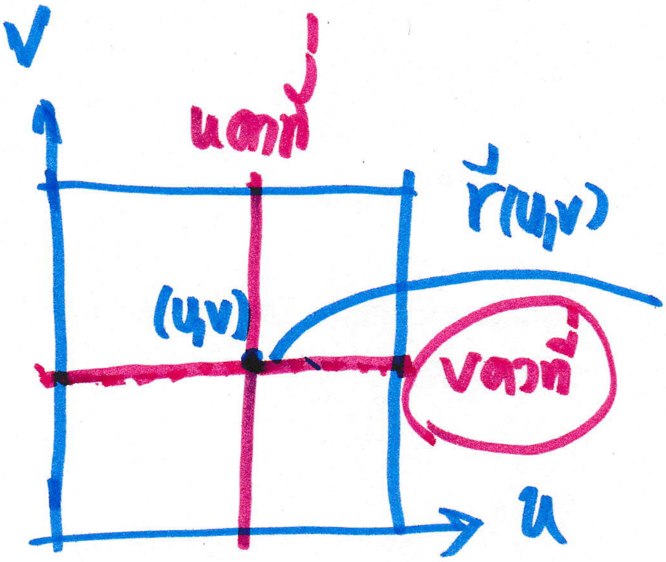
$$\frac{x^2}{\rho_0^2}+\frac{y^2}{\rho_0^2}+\frac{z^2}{\rho_0^2}=1$$

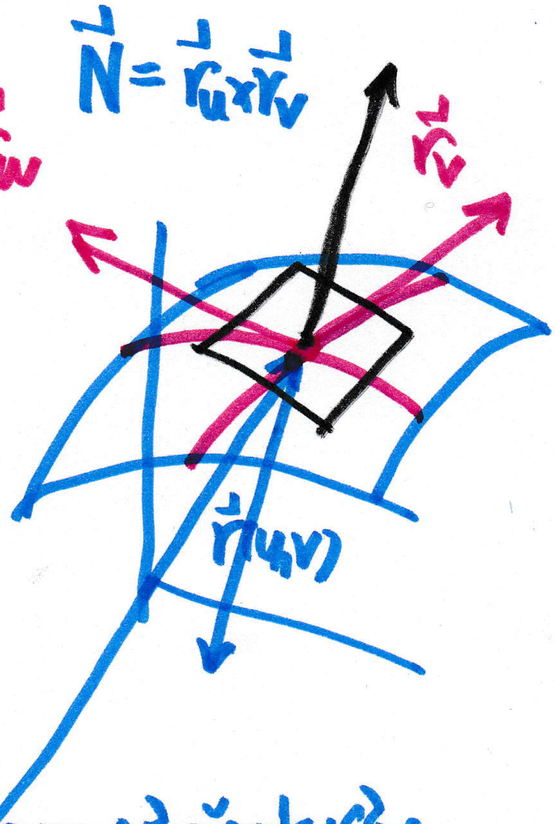
ทรงกลม $\vec{r}(u,v)=\rho_0\sin v\cos u\,\vec{i}+\rho_0\sin v\sin u\,\vec{j}+\rho_0\cos v\,\vec{k}$

ทรงรี $\vec{r}(u,v)=\underbrace{a\sin v\cos u}_{x}\,\vec{i}+\underbrace{b\sin v\sin u}_{y}\,\vec{j}+\underbrace{c\cos v}_{z}\,\vec{k}$

* $\leq u \leq$ $\leq v \leq$

Check! $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$?





S ที่เป็น $\vec{r}(u,v)$. (สมการแบบอิงตัวแปรเสริม.

Normal vector

$$\vec{N} = \vec{r}_u \times \vec{r}_v$$

$$\vec{n} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|}$$

$\Bigg\}$ เวกเตอร์ตั้งฉาก

$\pm \, \vec{r}_u \times \vec{r}_v$

Normal vector
① $\vec{r}(u,v)$
② $\vec{N} = \vec{r}_u \times \vec{r}_v$

$$\vec{r}(u,v) = a\sin v\cos u\,\vec{i} + a\sin v\sin u\,\vec{j} + \underline{a\cos v}\,\vec{k}$$

$$\vec{r}_u = -a\sin v\sin u\,\vec{i} + a\sin v\cos u\,\vec{j} + \vec{0}$$

$$\vec{r}_v = a\cos v\cos u\,\vec{i} + a\cos v\sin u\,\vec{j} - a\sin v\,\vec{k}$$

เวกเตอร์ตั้งฉาก $\vec{N}$

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a\sin v\sin u & a\sin v\cos u & 0 \\ a\cos v\cos u & a\cos v\sin u & -a\sin v \end{vmatrix}$$

$$= -a^2\sin^2 v\cos u\,\vec{i} - a^2\sin^2 v\sin u\,\vec{j}$$

$$\left(\underline{-a^2\sin v\cos v}\sin^2 u \; \underline{-a^2\sin v\cos v}\cos^2 u\right)\vec{k}$$

$$= -a^2\sin^2 v\cos u\,\vec{i} - a^2\sin^2 v\sin u\,\vec{j} - a^2\sin v\cos v\,\vec{k}$$

$$\therefore \vec{n} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|} = -\cos u\,\vec{i} - \sin u\,\vec{j} - \cos v\,\vec{k}$$

$$\vec{r}(u,v) = u\vec{i} + v\vec{j} + (1-u-v)\vec{k}$$

① Vector ตั้งฉาก $\quad \vec{r}_u = \vec{i} - \vec{k}$

$$\vec{r}_v = \vec{j} - \vec{k}$$

$$\vec{N} = \vec{r}_u \times \vec{r}_v = \left|\begin{array}{ccc} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{array}\right| \begin{array}{cc} \vec{i} & \vec{j} \\ 1 & 0 \\ 0 & 1 \end{array}$$

$$= \vec{k} - (-\vec{i} - \vec{j}) = \vec{i} + \vec{j} + \vec{k}$$

② $\quad |\vec{r}_u \times \vec{r}_v| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$$\vec{n} = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k}) \quad \#$$