ฟังก์ชันพหุนาม (polynomial function)

$$p(x) = x^2 + 2x + 5$$

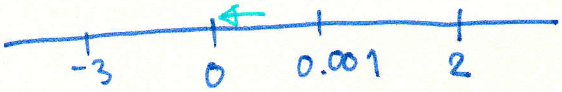
$$\lim_{x\to 1} p(x) = \lim_{x\to 1}(x^2+2x+5) = 1^2 + 2(1) + 5 = 8$$

ฟังก์ชันตรรกยะ (rational function)

$$f(x) = \frac{p(x)}{q(x)} \quad ; \; q(x) \neq 0$$

โดยที่ $p(x), q(x)$ เป็นพหุนาม.

Ex $$\lim_{x\to 0^+} \frac{\overset{+}{(x+3)}\overset{-}{(x-2)}}{\underset{+}{x}\underset{-}{(x-0.001)}} = +\infty$$



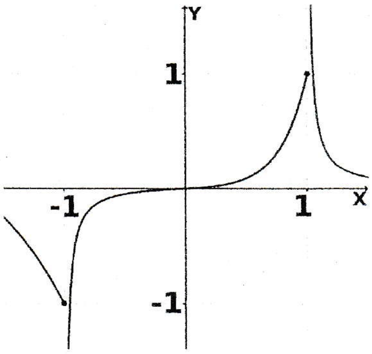
$x = 0.0001$

แบบทดสอบย่อย

เพื่อเช็คชื่อเข้าชั้นเรียน ประจำวันอังคารที่ 8 มกราคม พ.ศ.2562

ชื่อ-สกุล.. รหัสนักศึกษา............................ ลำดับที่...........

1. กำหนดให้ f นิยามดังภาพ ให้หาลิมิตต่อไปนี้ โดยเติมคำตอบลงในช่องว่าง



(a) $\lim\limits_{x\to -1^-} f(x) =$ ....

(b) $\lim\limits_{x\to -1^+} f(x) =$ ....

(c) $\lim\limits_{x\to 0^-} f(x) =$ ....

(d) $\lim\limits_{x\to 0^+} f(x) =$ ....

(e) $\lim\limits_{x\to 1^-} f(x) =$ ....

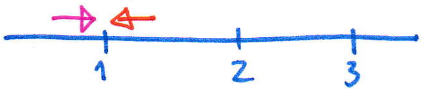
(f) $\lim\limits_{x\to 1^+} f(x) =$ ....

(g) The vertical asymtotes of the graph of f (เส้นกำกับแนวยืนของกราฟ f) is ....

2. ให้หาลิมิตต่อไปนี้

(a) $\lim\limits_{x\to 1^-} \dfrac{x-3}{(x-1)(x-2)^2} =$ $+\infty$

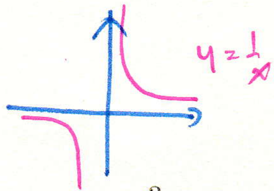
(b) $\lim\limits_{x\to 1^+} \dfrac{x-3}{(x-1)(x-2)^2} =$ $-\infty$



(c) $\lim\limits_{x\to 1} \dfrac{x-1}{(x-3)(x-2)^2} =$ 0

(d) $\lim\limits_{x\to 1} \dfrac{x+1}{(x-3)(x-2)^2} =$ $\dfrac{2}{-2} = -1$

$$\lim_{x\to 0^+} \frac{1}{x} = +\infty \qquad \lim_{x\to 0^-} \frac{1}{x} = -\infty$$



Example 1.10 Find

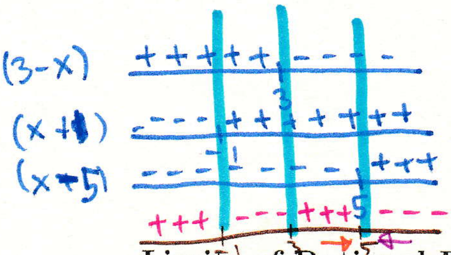
(a) $\displaystyle\lim_{x\to 5^+} \frac{(3-x)}{(x-5)(x+1)}$ (b) $\displaystyle\lim_{x\to 5^-} \frac{3-x}{(x-5)(x+1)}$ (c) $\displaystyle\lim_{x\to 5} \frac{3-x}{(x-5)(x+1)}$

Solution.

(a) $\displaystyle\lim_{x\to 5^+} \frac{3-x}{(x-5)(x+1)} = -\infty$

(b) $\displaystyle\lim_{x\to 5^-} \frac{3-x}{(x-5)(x+1)} = +\infty$

(c) $\displaystyle\lim_{x\to 5} \frac{3-x}{(x-5)(x+1)} = \text{DNE}$



Limits of Rational Functions $\dfrac{f(x)}{g(x)}$ **as** $x \to a$ **when** $\lim\limits_{x\to a} f(x) = 0 = \lim\limits_{x\to a} g(x)$ $\boxed{\frac{0}{0}}$

Example 1.11 Find

(a) $\displaystyle\lim_{x\to 5} \frac{x^2-10x+25}{x-5}$ (b) $\displaystyle\lim_{x\to 2} \frac{3x-6}{x^2-x-2}$ (c) $\displaystyle\lim_{x\to 3} \frac{x^2-5x+6}{x^2-6x+9}$.

Solution.

(a) $\displaystyle\lim_{x\to 5} \frac{x^2-10x+25}{x-5}$; $\displaystyle\lim_{x\to 5} f(x) = \lim_{x\to 5} x^2-10x+25 = 0$

$\displaystyle\lim_{x\to 5} g(x) = \lim_{x\to 5} x-5 = 0$

$$\lim_{x\to 5} \frac{(x-5)\cancel{(x-5)}}{\cancel{x-5}} = \lim_{x\to 5} (x-5) = 0$$

(b) $$\lim_{x\to 2} \frac{3x-6}{x^2-x-2} = \lim_{x\to 2} \frac{3\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} = \lim_{x\to 2} \frac{3}{x+1} = \frac{3}{3} = 1$$

(c) $$\lim_{x\to 3} \frac{x^2-5x+6}{x^2-6x+9} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}(x-3)} = \lim_{x\to 3} \frac{(x-2)}{(x-3)}$$

แยกลิมิต 2 ข้าง

$\displaystyle\lim_{x\to 3^+} \frac{x-2}{x-3} = +\infty$

$\displaystyle\lim_{x\to 3^-} \frac{x-2}{x-3} = -\infty$

$\therefore \displaystyle\lim_{x\to 3} \frac{x-2}{x-3} = \text{DNE}$

Theorem 1.4 Let

$$f(x) = \frac{p(x)}{q(x)}$$

be a rational function, and let a be any real number.

(a) If $q(a) \neq 0$, then $\lim_{x \to a} f(x) = f(a)$.

(b) If $q(a) = 0$ but $p(a) \neq 0$, then $\lim_{x \to a} f(x)$ does not exist.

Limits Involving Radicals

(ควก)

$a^2 - b^2 = (a-b)(a+b)$

Example 1.12 Find $\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2}$.

Solution.

$\boxed{\text{Solution 1}}$

$$\lim_{x \to 4} \frac{(\sqrt{x})^2 - (2)^2}{\sqrt{x} - 2} = \lim_{x \to 4} \frac{\cancel{(\sqrt{x} - 2)}(\sqrt{x} + 2)}{\cancel{\sqrt{x} - 2}}$$

$$= \lim_{x \to 4} (\sqrt{x} + 2)$$

$$= \sqrt{4} + 2 = 2 + 2 = 4$$

$\boxed{\text{Solution 2}}$

$$\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} = \lim_{x \to 4} \frac{x - 4}{(\sqrt{x} - 2)} \cdot \frac{(\sqrt{x} + 2)}{(\sqrt{x} + 2)}$$

Conjugate ของ $\sqrt{x} - 2$ คือ $\sqrt{x} + 2$

$$= \lim_{x \to 4} \frac{(x - 4)(\sqrt{x} + 2)}{(\sqrt{x})^2 - (2)^2}$$

$$= \lim_{x \to 4} \frac{\cancel{(x - 4)}(\sqrt{x} + 2)}{\cancel{x - 4}}$$

$$= \lim_{x \to 4} \sqrt{x} + 2$$

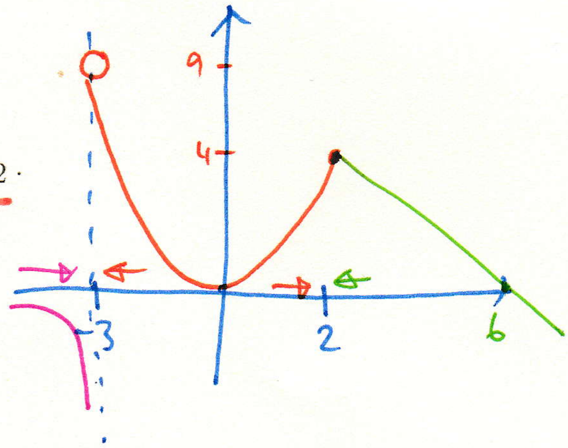
$$= 4$$

\#

Limits of Piecewise-Defined Functions (ฟังก์ชันที่นิยามเป็นช่วงๆ)

Example 1.13 Let

$$f(x) = \begin{cases} \dfrac{2}{x+3}, & x < -3 \\ x^2, & -3 < x \le 2 \\ 6 - x, & x > 2 \end{cases}$$



Find

(a) $\lim_{x \to -3} f(x)$ (b) $\lim_{x \to 0} f(x)$ (c) $\lim_{x \to 2} f(x)$.

Solution.

(a) $\lim_{x \to -3} f(x)$ พิจารณา $\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \dfrac{2}{x+3} = -\infty$

$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} x^2 = 9$

$\therefore \lim_{x \to -3} f(x)$ DNE.

(b) $\lim_{x \to 0} f(x) = \lim_{x \to 0} x^2 = 0$

(c) $\lim_{x \to 2} f(x)$ พิจารณา $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 = 4$

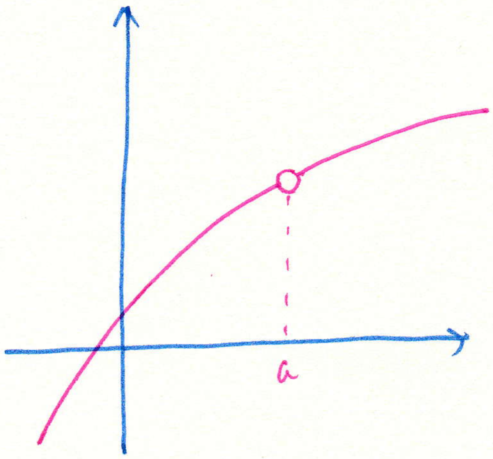
$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 6 - x = 4$

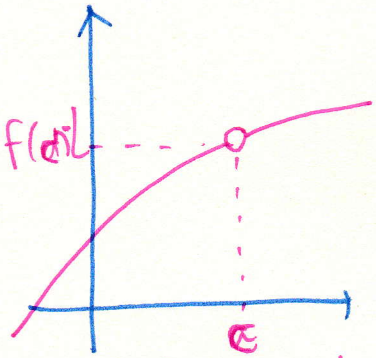
$\therefore \lim_{x \to 2} f(x) = 4$

ความต่อเนื่องของฟังก์ชัน

Continuity of function.

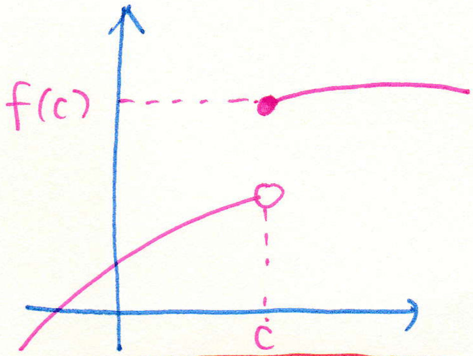
(ความไม่ต่อเนื่อง : discontinuity)





removable discontinuity

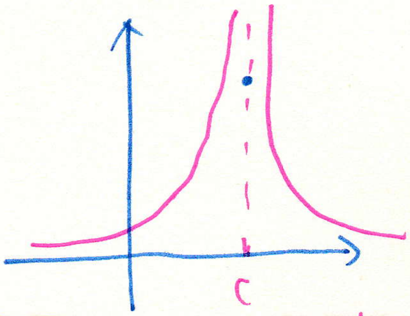
f(c) หาค่าไม่ได้



jump discontinuity

f(c) หาค่าได้

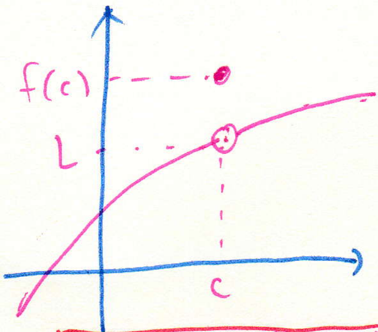
$\lim_{x \to c} f(x)$ ไม่มี



infinite discontinuity

f(c) ไม่มี

$\lim_{x \to c} f(x)$ ไม่มีค่า (ไม่เป็นจำนวนจริง)



removable discontinuity

f(c) หาค่าได้

$\lim_{x \to c} f(x) = L$

แต่ $f(c) \neq \lim_{x \to c} f(x)$

1.3 Continuity

Definition A function f is said to be ***continuous at $x = c$*** provided the following conditions are satisfied:

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$.

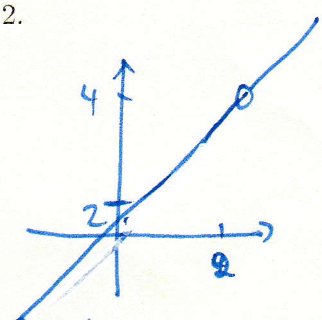
Example 1.14 Determine whether the following functions are continuous at $x = 2$.

$$f(x) = \frac{x^2-4}{x-2}, \qquad g(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ 2, & x = 2, \end{cases} \qquad h(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ 4, & x = 2. \end{cases}$$

Solution. $\boxed{1}$ $f(x) = \dfrac{x^2-4}{x-2}$ (1) $f(2)$ หาค่าไม่ได้

$D_f = \mathbb{R} - \{2\}$

$\therefore f(x)$ ไม่ต่อเนื่องที่ $x=2$



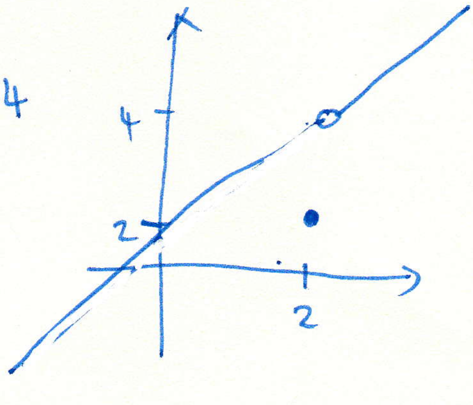
$\boxed{2}$ $g(x) = \begin{cases} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} & ; x \neq 2 \\ 2 & ; x = 2 \end{cases} \Rightarrow g(x) = \begin{cases} x+2 & ; x \neq 2 \\ 2 & ; x = 2 \end{cases}$

(1) $g(2) = 2$

(2) $\lim_{x \to 2} g(x) = \lim_{x \to 2} x+2 = 4$

(3) $\lim_{x \to 2} g(x) \neq g(2)$

$\therefore f$ ไม่ต่อเนื่องที่ $x=2$



$\boxed{3}$ $h(x) = \begin{cases} x+2 & ; x \neq 2 \\ 4 & ; x = 2 \end{cases}$

(1) $h(2) = 4$

(2) $\lim_{x \to 2} h(x) = \lim_{x \to 2} x+2 = 4$

(3) $h(2) = 4 = \lim_{x \to 2} h(x)$

$\therefore f$ ต่อเนื่องที่ $x=2$