ห้องทำแบบฝึกหัด ทุกวันพุธ

10.00-12.00
13.00-15.00
17.30-19.30

} ห้อง MB2217

เริ่ม พุธที่ 6 ก.พ. 2562

ให้ u เป็นฟังก์ชันของ x.

สูตรทั่วไป

1. $\dfrac{dc}{dx} = 0$

2. $\dfrac{du^n}{dx} = nu^{n-1}\dfrac{du}{dx}$

Ex $\dfrac{d}{dx}(\overset{u}{\boxed{2x^2+4}})^2 = 2(2x^2+4)\dfrac{d}{dx}(2x^2+4)$

$= 2(2x^2+4)(4x)$

3. $\dfrac{d(u \cdot v)}{dx} = u\dfrac{dv}{dx} + v \cdot \dfrac{du}{dx}$

4. $\dfrac{d}{dx}(u \pm v) = \dfrac{du}{dx} \pm \dfrac{dv}{dx}$

5. $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{(v)^2}$

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6. $\dfrac{d}{dx}(\sin u) = \cos u \dfrac{du}{dx}$

Ex $\dfrac{d}{dx}(\cos \overset{u}{\boxed{(x^2+4)}}) = -\sin(x^2+4)\dfrac{d}{dx}(x^2+4)$

$= -\sin(x^2+4)(2x)$

$= -2x\sin(x^2+4)$

7. $\dfrac{d}{dx}(\cos u) = -\sin u \dfrac{du}{dx}$

8. $\dfrac{d}{dx}(\tan u) = \sec^2 u \dfrac{du}{dx}$

Ex $\dfrac{d}{dx}(\cos \overset{u}{\boxed{(\sin x)}}) = -\sin(\sin x)\dfrac{d}{dx}(\sin x)$

$= -[\sin(\sin x)](\cos x)$

9. $\dfrac{d}{dx}(\cot u) = -\operatorname{cosec}^2 u \dfrac{du}{dx}$

10. $\dfrac{d}{dx}(\sec u) = \sec u \tan u \dfrac{du}{dx}$

11. $\dfrac{d}{dx}(\operatorname{cosec} u) = -\operatorname{cosec} u \cot u \dfrac{du}{dx}$

HW

$$\frac{1}{\cos x} = \sec x$$

$$\therefore \frac{1}{\cos^2 x} = \sec^2 x$$

Example 2.25 Find $\dfrac{dg}{dt}$ if $g(t) = \dfrac{-3}{\cos^2(t-7)}$ $\longrightarrow$ $g(t) = -3\sec^2(t-7)$

$g(t) = -3(\boxed{\sec(t-7)})^2$

Solution.

$$\frac{dg}{dt} = -6\sec(t-7)\frac{d}{dt}(\sec(t-7))$$

$$= -6\sec(t-7)\sec(t-7)\tan(t-7)\frac{d}{dt}(t-7)$$

$$= -6\sec^2(t-7)\tan(t-7) \quad \#$$

Example 2.26 Find $\dfrac{ds}{dt}$ if $s(t) = \sec^3(\cos(\sqrt{t}))$ $= (\boxed{\sec(\cos(\sqrt{t}))})^3$

Solution.

$$\frac{ds}{dt} = 3\sec^2(\cos\sqrt{t})\frac{d}{dt}(\sec\boxed{\cos\sqrt{t}})$$

$$= 3\sec^2(\cos\sqrt{t})\sec(\cos\sqrt{t})\tan(\cos\sqrt{t})\frac{d}{dt}[\cos\sqrt{t}]$$

$$= 3\sec^3(\cos\sqrt{t})\tan(\cos\sqrt{t})(-\sin\sqrt{t})\frac{d\sqrt{t}}{dt}$$

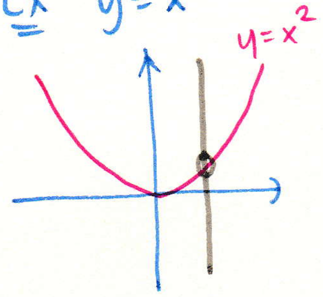
$$= 3\sec^3(\cos\sqrt{t})\tan(\cos\sqrt{t})(-\sin\sqrt{t})\left(\frac{1}{2\sqrt{t}}\cdot\frac{dt}{dt}\right)$$

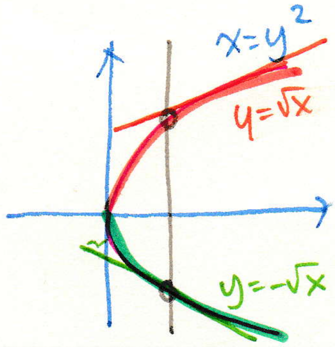
$$= 3\sec^3(\cos\sqrt{t})\tan(\cos\sqrt{t})(-\sin\sqrt{t})\left(\frac{1}{2\sqrt{t}}\right) \quad \#$$

$$\boxed{\begin{array}{l} \dfrac{d\sqrt{x}}{dx} = \dfrac{1}{2\sqrt{x}} \\[2ex] \dfrac{d\sqrt{u}}{dx} = \dfrac{1}{2\sqrt{u}}\dfrac{du}{dx} \end{array}}$$

$y = \sqrt{\dfrac{x^2+1}{x^2+4}}$

Ex $y = x^2$





3 Topics of Differentiation

→ Explicit Function [ฟังก์ชันชัดแจ้ง]

3.1 Implicit Function [ฟังก์ชันแฝง / ฟังก์ชันโดยปริยาย]

Sometimes we deal with an expression that is not explicitly given but instead implicitly given in the form $f(x,y) = 0$ or $f(x,y) = g(x,y)$, e.g., $2y^2 = 3x + 8$. To find the derivative in this case,

1. first differentiate both sides of the expression,
2. then solve for $\dfrac{dy}{dx}$.

Example 3.1 Find $\dfrac{dy}{dx}$ where $\boxed{y^2 - x + 1 = 0}$. [y เป็นฟังก์ชันของ x]

Solution. 1 diff ทั้งสองข้าง : $\dfrac{d}{dx}[y^2 - x + 1] = \dfrac{d}{dx}[0]$

$$2y\frac{dy}{dx} - 1 + 0 = 0$$

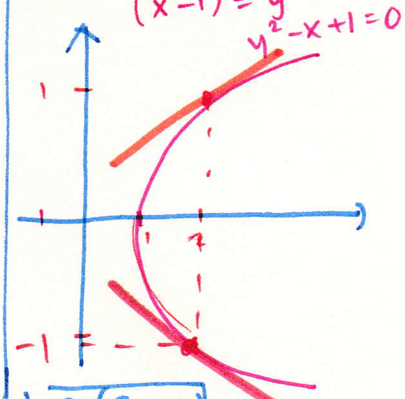
$$2y\boxed{\frac{dy}{dx}} = 1$$

2 แก้สมการหา $\dfrac{dy}{dx}$: $\boxed{\dfrac{dy}{dx} = \dfrac{1}{2y}}$

$y^2 - x + 1 = 0$

$\Downarrow$ $x = y^2 + 1$

$(x-1) = y^2$



จุด (2,−1)

เส้นสัมผัสกราฟ มีความชันคือ

$\therefore \left.\dfrac{dy}{dx}\right|_{(2,-1)} = \dfrac{1}{2(-1)} = -\dfrac{1}{2}$

สมการคือ $y-(-1) = (-\frac{1}{2})(x-2)$

$\therefore y+1 = (-\frac{1}{2})(x-2)$

จุด (2,1)

เส้นสัมผัสกราฟ มีความชันคือ

$\left.\dfrac{dy}{dx}\right|_{(2,1)} = \dfrac{1}{2(1)} = \dfrac{1}{2}$

สมการคือ $y-2 = \frac{1}{2}(x-2)$

Example 3.2 Find $\dfrac{dy}{dx}$ where $4xy^2 = 9$.

Solution. 1 diff ทั้งสองข้าง : $\dfrac{d}{dx}[4xy^2] = \dfrac{d}{dx}(9)$

$$4\left[x\frac{dy^2}{dx} + y^2\frac{dx}{dx}\right] = 0$$

$$4\left[(x)(2y)\frac{dy}{dx} + y^2\right] = 0$$

$$2xy\frac{dy}{dx} + y^2 = 0$$

$$2xy\boxed{\frac{dy}{dx}} = -y^2$$

2 แก้สมการหา $\dfrac{dy}{dx}$: $\boxed{\dfrac{dy}{dx} = \dfrac{-y^2}{2xy}}$

Example 3.3 Find $\frac{dy}{dx}$ where $4x^4y - y^3 = 4\sin y$.

Solution. (1) $\frac{d}{dx}[4x^4y - y^3] = \frac{d}{dx}[4\sin y]$

$4[x^4\frac{d}{dx}y + y\frac{d}{dx}x^4] - 3y^2\frac{dy}{dx} = 4\cos y\frac{dy}{dx}$

$4[x^4\frac{dy}{dx} + 4x^3y] - 3y^2\frac{dy}{dx} = 4\cos y\frac{dy}{dx}$

$4x^4\boxed{\frac{dy}{dx}} + 16x^3y - 3y^2\boxed{\frac{dy}{dx}} = 4\cos y\boxed{\frac{dy}{dx}}$

$\boxed{2} \to \quad 4x^4\frac{dy}{dx} - 3y^2\frac{dy}{dx} - 4\cos y\frac{dy}{dx} = -16x^3y$

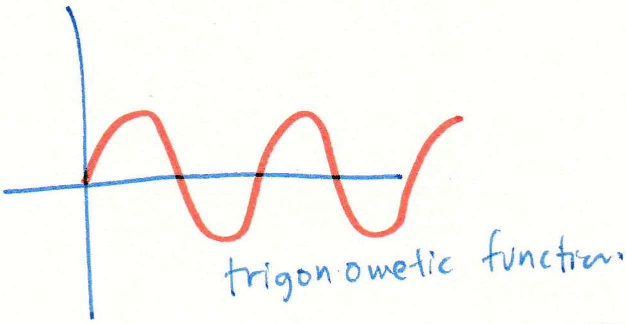
$(4x^4 - 3y^2 - 4\cos y)\boxed{\frac{dy}{dx}} = -16x^3y$

$\therefore \boxed{\frac{dy}{dx} = -\frac{16x^3y}{4x^4 - 3y^2 - 4\cos y}}$

X **Example 3.4** Find the slopes of the tangent lines to the curve $y^2 - x + 1 = 0$ at the points $(2, -1)$ and $(2, 1)$.

Solution.

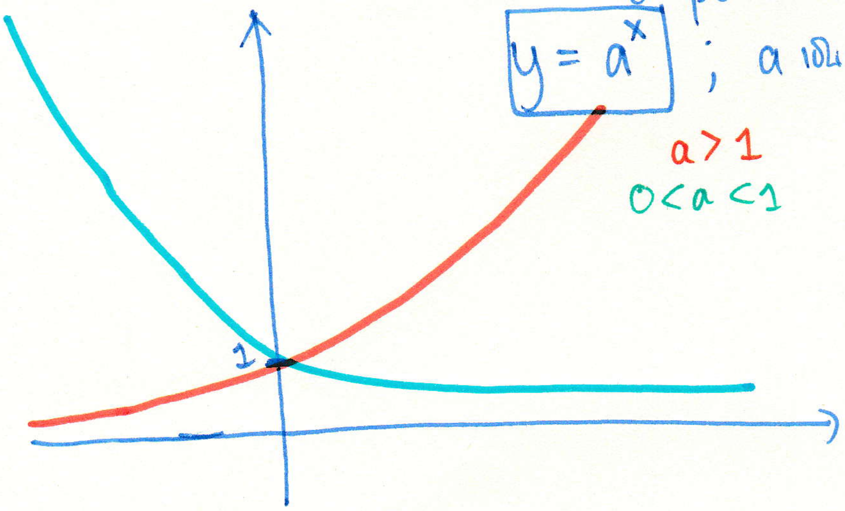
HW. Ex เพิ่มเติม หา $\frac{dy}{dx}$ เมื่อ $(x, y) = (1, 1)$ โดยที่ $x^2 + y^2 = 2$



trigonometric function

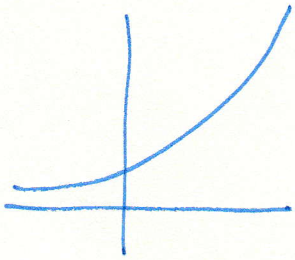
Exponential and logarithmic function

$\boxed{y = a^x}$; a เป็น constant ซึ่ง $a \neq 1, a > 0$



ค่า e → $e \approx 2.71$

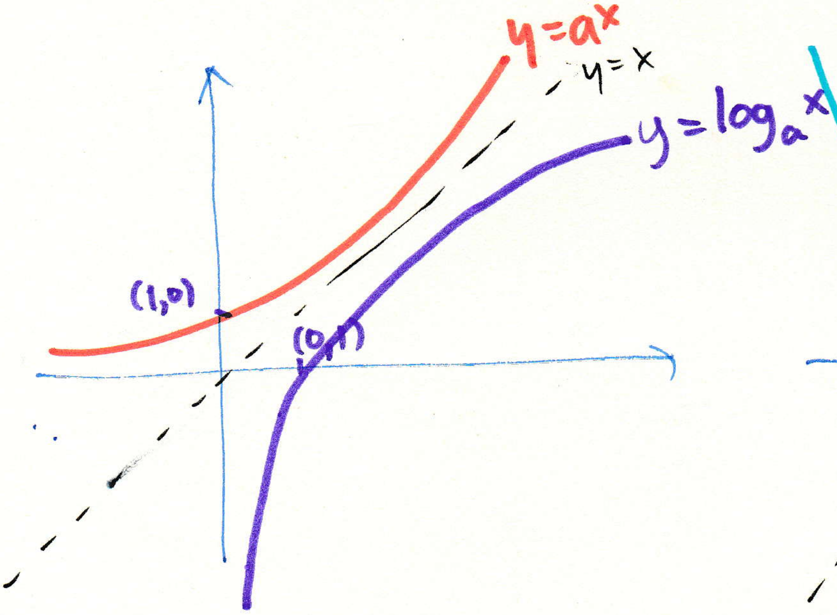
$\boxed{y = e^x}$



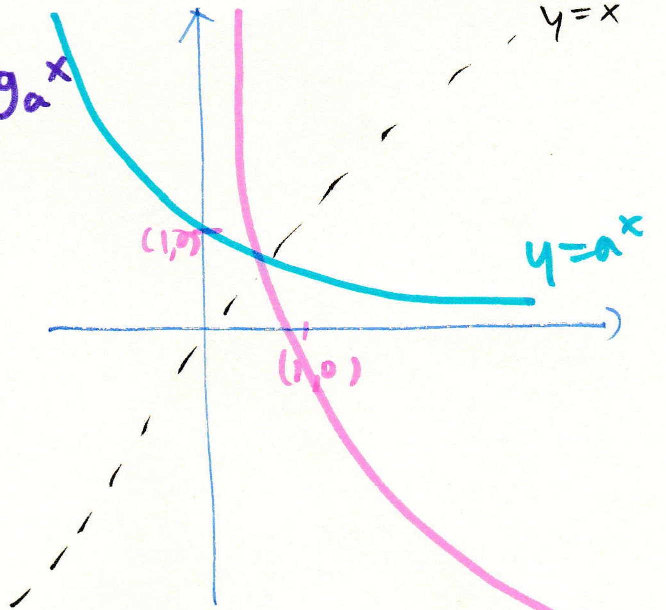
อินเวอร์สของฟังก์ชัน

$y = a^x \longleftrightarrow \boxed{x = a^y \Longleftrightarrow y = \log_a x} \Longrightarrow \log_{ฐาน} x = ตัวเลข$

$x = ฐาน^{ตัวเลข}$



$a > 0$



$0 < a < 1$ $y = \log_a x$

$\boxed{y = \ln x}$

$y = \log_e x$

3.2 Derivatives of Exponential and Logarithmic Functions

3.2.1 Derivatives of Exponential and Logarithmic Functions

Here is a summary of the derivative formulas in this section.

THEOREM 3.1

1. $\frac{d}{dx}[a^x] = a^x \ln a$
2. $\frac{d}{dx}[e^x] = e^x$
3. $\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$; $x > 0$
4. $\frac{d}{dx}[\ln x] = \frac{1}{x}$; $x > 0$

Example 3.5 Find the derivative of $y = 2^{\ln 5x}$.

Solution.

$$\frac{d}{dx}\left(2^{\overset{u}{\boxed{\ln 5x}}}\right) = 2^{\ln 5x} \ln 2 \frac{d}{dx}(\ln 5x)$$

$$= 2^{\ln 5x} \ln 2 \left(\frac{1}{5x}\right) \frac{d(5x)}{dx}$$

$$= \frac{2^{\ln 5x} \ln 2 (\cancel{5})}{\cancel{5}x}$$

$$= \frac{5 \ln 5 \cdot 2^{\ln 5x}}{x} \quad \#$$

Example 3.6 Find the derivative of $y = e^{\cos^3 x}$.

Solution.

$$\frac{d}{dx}\left(e^{\boxed{\cos^3 x}}\right) = e^{\cos^3 x} \frac{d}{dx}(\cos^3 x)$$

$$= e^{\cos^3 x}\, 3\cos^2 x \frac{d}{dx}(\cos x)$$

$$= e^{\cos^3 x} \cdot 3\cos^2 x(-\sin x) \quad \#$$

$$= -3 \sin x \cos^2 x\, e^{\cos^3 x} \quad \#$$

HW **Example 3.7** Find the derivative of $y = (\log_3 x)^3$.

Solution.