u, v เป็นฟังก์ชันของ x

① $\frac{d}{dx}[c] = 0$

② $\frac{d}{dx}[u \pm v] = \frac{du}{dx} \pm \frac{dv}{dx}$

③ $\frac{du^n}{dx} = nu^{n-1}$; u เป็นฟังก์ชัน, n เป็นตัวเลข.

④ $\frac{d(u \cdot v)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

⑤ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

⑥ $\frac{d}{dx}[\sqrt{u}] = \frac{1}{2\sqrt{u}}\frac{du}{dx}$

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⑦ $\frac{d}{dx}[\sin u] = \cos u \frac{du}{dx}$

⑧ $\frac{d}{dx}[\cos u] = -\sin u \frac{du}{dx}$

⑨ $\frac{d}{dx}[\tan u] = \sec^2 u \frac{du}{dx}$

⑩ $\frac{d}{dx}[\cot u] = -\mathrm{cosec}^2 u \frac{du}{dx}$

⑪ $\frac{d}{dx}[\sec u] = \sec u \tan u \frac{du}{dx}$

⑫ $\frac{d}{dx}[\mathrm{cosec}\, u] = -\mathrm{cosec}\, u \cot u \frac{du}{dx}$

---

⑬ $\frac{d}{dx}[a^u] = a^u \ln a \frac{du}{dx}$

⑭ $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

⑮ $\frac{d}{dx}[\log_a u] = \frac{1}{u \ln a}\frac{du}{dx}$

⑯ $\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx}$

Ex 7 $y = (\boxed{\log_3 x})^3$ หา y'

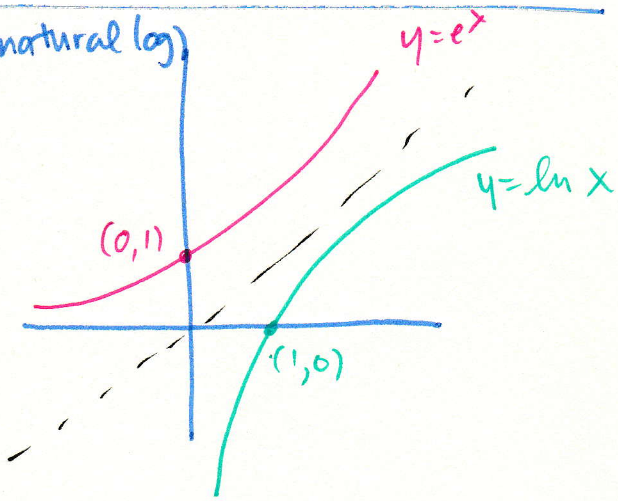
$\boxed{y = ♡^3}$

$\therefore y' = 3(\log_3 x)^2 \frac{d}{dx}[\log_3 x]$

$= 3(\log_3 x)^2 \cdot \frac{1}{x \ln 3}$ #

สมบัติของฟังก์ชันลอการิทึม

(ดู natural log)



1. $\ln 1 = 0$
2. $\ln(A \cdot B) = \ln A + \ln B$
3. $\ln\left(\frac{A}{B}\right) = \ln A - \ln B$
4. $\ln A^n = n \ln A$

ข้อควรระวัง

ห้ามทำ XXX

1. $\ln(A+B) \neq \ln A + \ln B$
2. $\ln(A \cdot B) \neq \ln A \cdot \ln B$
3. $\ln\left(\frac{A}{B}\right) \neq \frac{\ln A}{\ln B}$
4. $(\ln A)^n \neq n \ln A$

การใช้ลอการิทึมช่วยหาอนุพันธ์

1. take ln (ใช้ log ฐานไหนก็ได้ โดยทั่วไปใช้ ln ฐาน e)
2. ใช้สมบัติของ logarithm ช่วยกระจายเทอมให้ง่ายขึ้น
3. diff ทั้งสองข้าง

3.2.2 Logarithm Differentiation

การใช้ลอการิทึมช่วยในการหาอนุพันธ์

We now consider a technique called logarithmic differentiation. This technique is useful for functions that are composed of products, quotients, and powers.

Example 3.8 Find the derivative of $y = \dfrac{x \ln x \sqrt{x^2+1}}{(1+x^4)^2}$. ; $x > 0$

Solution. $\boxed{1}$ Take ln ทั้งสองข้าง

$$\ln y = \ln\left(\frac{\boxed{x \ln x \sqrt{x^2+1}}}{\boxed{(1+x^4)^2}}\right)$$

$\boxed{2}$
$$\ln y = \boxed{\ln x + \ln(\ln x) + \ln(\sqrt{x^2+1})} - \boxed{\ln(1+x^4)^2}$$
$$\ln y = \ln x + \ln(\ln x) + \ln(x^2+1)^{\frac{1}{2}} - \ln(1+x^4)^2$$
$$\ln y = \ln x + \ln(\ln x) + \frac{1}{2}\ln(x^2+1) - 2\ln(1+x^4)$$

$\boxed{3}$ diff;
$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[\ln x] + \frac{d}{dx}[\ln(\ln x)] + \frac{1}{2}\frac{d}{dx}\ln(x^2+1) - 2\frac{d}{dx}[\ln(1+x^4)]$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{\ln x}\cdot\frac{1}{x} + \frac{1}{\cancel{2}}\cdot\frac{1}{x^2+1}(\cancel{2}x) - 2\cdot\frac{1}{1+x^4}(4x^3)$$
$$\frac{dy}{dx} = y\left[\frac{1}{x} + \frac{1}{x\ln x} + \frac{x}{x^2+1} - \frac{8x^3}{1+x^4}\right]$$

$$\therefore \frac{dy}{dx} = \left[\frac{x\ln x\sqrt{x^2+1}}{(1+x^4)^2}\right]\left[\frac{1}{x} + \frac{1}{x\ln x} + \frac{x}{x^2+1} - \frac{8x^3}{1+x^4}\right]$$

Example 3.9 Find the derivative of $y = (x^2+1)^{\sin x}$.

Solution. $\boxed{1}$ Take ln
$$\ln y = \ln\left[(x^2+1)^{\sin x}\right]$$

$\boxed{2}$
$$\ln y = \sin x \cdot \ln(x^2+1)$$

$\boxed{3}$ diff;
$$\frac{1}{y}\frac{dy}{dx} = \sin x\frac{d}{dx}(\ln(x^2+1)) + \ln(x^2+1)\frac{d}{dx}(\sin x)$$
$$\frac{1}{y}\frac{dy}{dx} = \sin x \cdot \frac{1}{x^2+1}(2x) + \ln(x^2+1)\cos x$$
$$\frac{dy}{dx} = y\left[\frac{2x\sin x}{x^2+1} + \cos x \cdot \ln(x^2+1)\right]$$
$$\frac{dy}{dx} = (x^2+1)^{\sin x}\left[\frac{2x\sin x}{x^2+1} + (\cos x)\ln(x^2+1)\right]$$

Ex เพิ่มเติม $y = x^x$

$y = x^n$

$y = a^x$

take ln;

1. $\ln y = \ln(x^x)$

2. $\ln y = x \ln(x)$

3. diff;

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\ln x) + \ln x \frac{dx}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \cancel{x}\cdot\frac{1}{\cancel{x}} + \ln x \cdot 1$$

$$\frac{dy}{dx} = y[1 + \ln x]$$

$$\boxed{\frac{dy}{dx} = x^x[1 + \ln x]}$$

#

$y = \sin x$ inverse ของฟังก์ชัน : $x = \sin y$

(สลับ x, สลับ y)

$$\downarrow$$

$$\boxed{y = \sin^{-1} x}$$

* $\sin^{-1} x \neq \boxed{(\sin x)^{-1} = \frac{1}{\sin x}}$

อินเวอร์สของ $y = \sin x$

→ ฟังก์ชันอินเวอร์สของ $y = \sin x$

คือ $y = \arcsin x$

3.3 Derivatives Of The Inverse Trigonometric Functions

Let $y = \sin^{-1} x$, then $\boxed{x = \sin y.}$ Hence,

หาอนุพันธ์ของฟังก์ชัน $y = \sin^{-1} x$

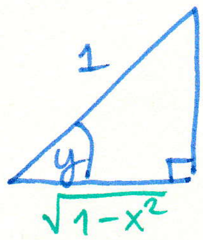
$y = \sin^{-1} x \Longleftrightarrow x = \sin y$ หา $\frac{dy}{dx}$

$\sin y = \frac{x}{1}$

diff ทั้งสองข้าง

$$\frac{d}{dx}(x) = \frac{d}{dx}(\sin y)$$

$$1 = \cos y \frac{dy}{dx}$$



$\cos y = \sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{1}{\cos y} \Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$$

The method used to derive this formula can be used to obtain generalized derivative formulas for the remaining inverse trigonometric functions. The following is a complete list of these formulas, each of which is valid on the natural domain of the function.

Theorem 3.2

1. $\frac{d}{dx}[\sin^{-1} u] = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$
2. $\frac{d}{dx}[\cos^{-1} u] = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$
3. $\frac{d}{dx}[\tan^{-1} u] = \frac{1}{1+u^2}\frac{du}{dx}$
4. $\frac{d}{dx}[\cot^{-1} u] = \frac{-1}{1+u^2}\frac{du}{dx}$
5. $\frac{d}{dx}[\sec^{-1} u] = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$
6. $\frac{d}{dx}[\csc^{-1} u] = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

HW

Example 3.10 Find the derivative of $y = \sin^{-1}(x^3 + 1)$.

Solution.

$$y' = \frac{1}{\sqrt{1-(x^3+1)^2}} \frac{d}{dx}(x^3+1)$$

$$= \frac{3x^2}{\sqrt{1-(x^3+1)^2}} \quad \#$$

Example 3.11 Find the derivative of $y = \cos^{-1}(\sqrt{x})$

Solution.

$$y' = -\frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx}(\sqrt{x})$$

$$= -\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

Example 3.12 Find $\frac{dy}{dx}$ when $x^3 + x\tan^{-1} y = y$.

Solution. diff ทั้งสองข้าง :

$$\frac{d}{dx}[x^3 + x\tan^{-1} y] = \frac{dy}{dx}$$

$$3x^2 + \left[x\frac{d}{dx}(\tan^{-1} y) + \tan^{-1} y \frac{dx}{dx}\right] = \frac{dy}{dx}$$

$$3x^2 + \left[x \cdot \frac{1}{1+y^2} \cdot \boxed{\frac{dy}{dx}} + \tan^{-1} y\right] = \boxed{\frac{dy}{dx}}$$

$$\frac{x}{1+y^2}\frac{dy}{dx} - \frac{dy}{dx} = -\tan^{-1} y - 3x^2$$

$$\left(\frac{x}{1+y^2} - 1\right)\boxed{\frac{dy}{dx}} = -(\tan^{-1} y + 3x^2)$$

$$\therefore \boxed{\frac{dy}{dx} = -\frac{\tan^{-1} y + 3x^2}{\left(\frac{x}{1+y^2} - 1\right)}}$$