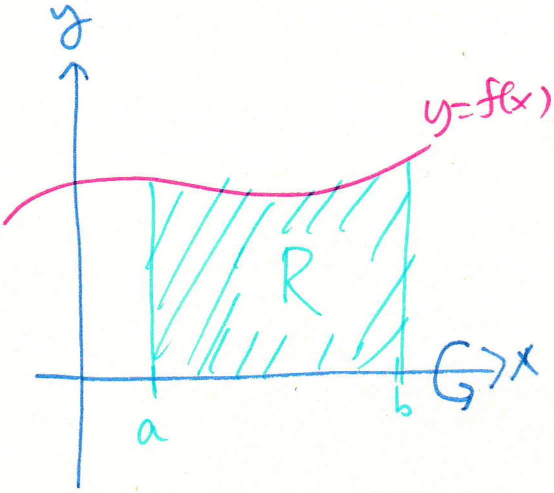
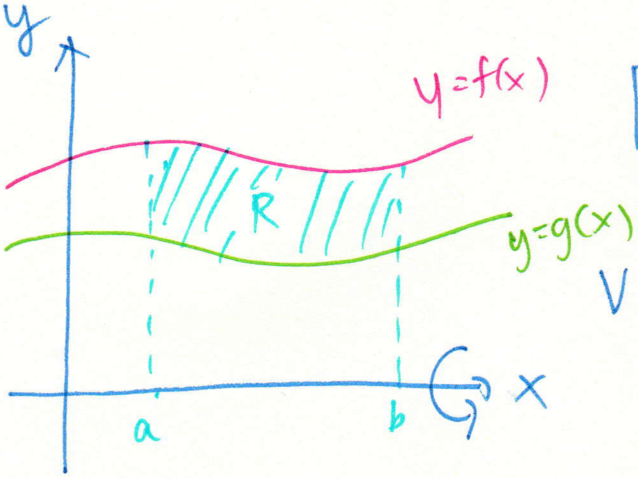


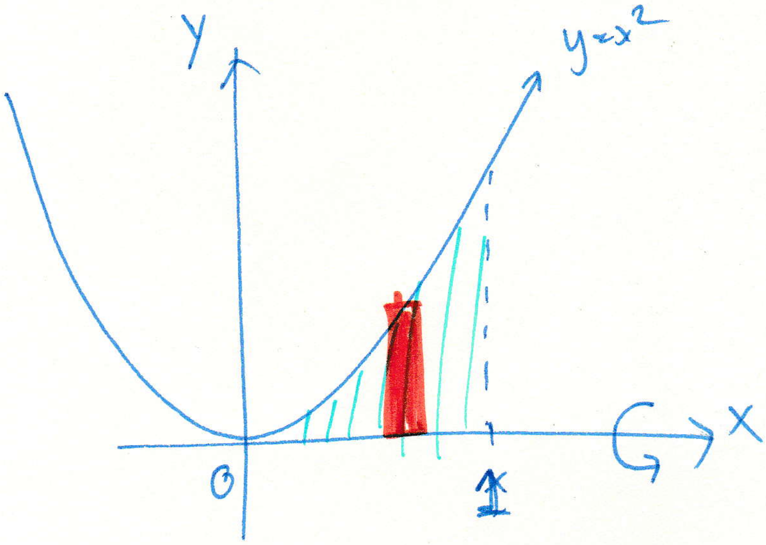
$\boxed{\text{Disk}}$

$$V=\int_a^b \pi (f(x))^2 dx$$

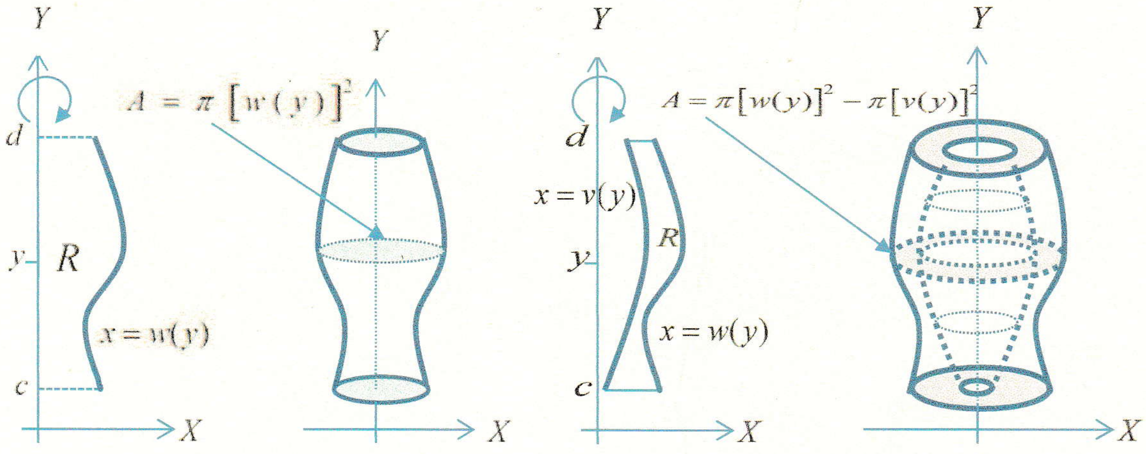


$\boxed{\text{Washer}}$

$$V=\pi\int_a^b [(f(x))^2-(g(x))^2]\,dx.$$



Volume by Disks and Washers perpendicular to the Y-axis



Disks Washers

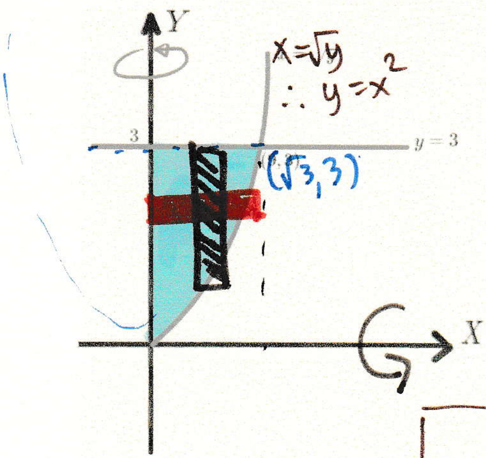
The methods of disks and washers have analogs for regions that are revolved about the Y-axis. Using the method of slicing and Formula (6.4), the following formulas for the volumes of the solid are

$$V = \int_c^d \pi[w(y)]^2 dy \qquad (disks), \tag{6.7}$$

$$V = \int_c^d \pi\left([w(y)]^2 - [v(y)]^2\right) dy \qquad (washers). \tag{6.8}$$

Example 6.8 Find the volume of the solid generated when the region enclosed by $x = \sqrt{y}$, $x = 0$, and $y = 3$ is revolved about the Y-axis.

$x^2 = y$



$$V = \int_0^3 [\pi(\sqrt{y})^2]\,dy$$

$$V = \pi\int_0^3 y\,dy$$

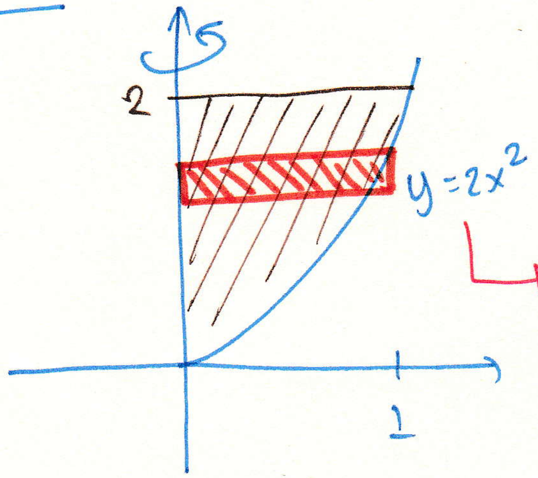
$$= \pi \frac{y^2}{2}\Big|_0^3 = \frac{9\pi}{2} \quad \#$$

ถ้าหมุนรอบแกน X

Washer

$$\boxed{V = \int_0^{\sqrt{3}} \pi[(3)^2 - (x^2)^2]\,dx}$$

ข้อสังเกต



disk

$$V = \int_0^2 \cancel{2\pi}\left(\cancel{2x}\right)^2 \, dy \quad \times$$

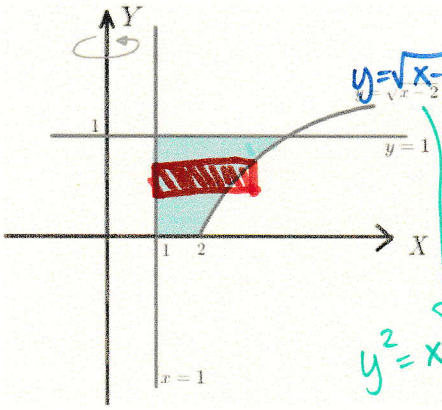
$y = 2x^2$

$\frac{y}{2} = x^2$

$x = \oplus\sqrt{\frac{y}{2}}$

$$\boxed{V = \int_0^2 \pi \left(\frac{\sqrt{y}}{\sqrt{2}}\right)^2 dy}$$

Example 6.9 Find the volume of the solid generated when the region enclosed by $x = 1$, $y = \sqrt{x-2}$, $y = 0$, and $y = 1$ is revolved about the Y-axis.



$y^2 = x-2$

$\therefore\ x = y^2+2$

หมุนรอบแกน y ใช้วิธี washer.

$$V = \pi\int_0^1 \left[(y^2+2)^2 - 1^2\right] dy$$

$$V = \pi\int_0^1 \left[y^4+4y^2+4-1\right] dy$$

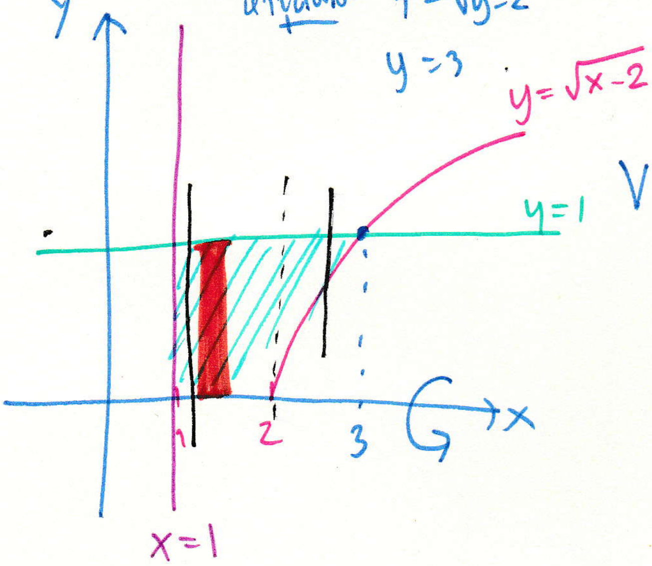
$$= \pi\int_0^1 (y^4+4y^2+3)\,dy$$

$$= \pi\left[\frac{y^5}{5}+\frac{4y^3}{3}+3y\right]_0^1 = \pi\left[\frac{1}{5}+\frac{4}{3}+3\right] = \frac{58\pi}{15}$$

หมุนรอบแกน x โดยวิธี disk + washer

หาจุดตัด $1 = \sqrt{y-2}$

$y = 3$



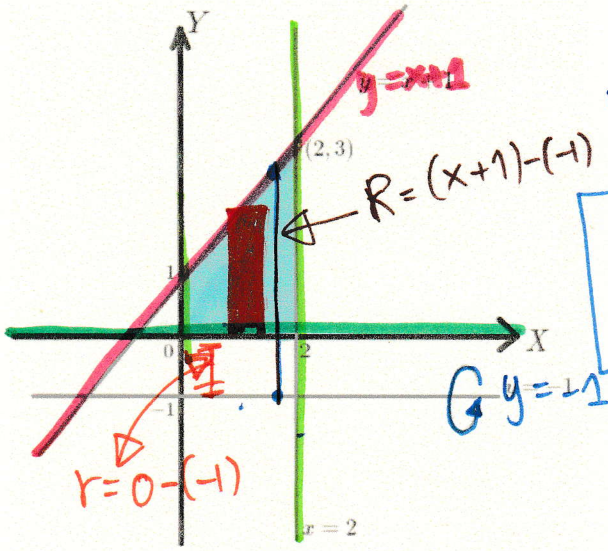
disk washer

$$V = \boxed{\int_1^2 \pi(1)^2\,dx} + \boxed{\pi\int_2^3 \left[1^2 - (\sqrt{x-2})^2\right] dx}$$

Other axes of revolution

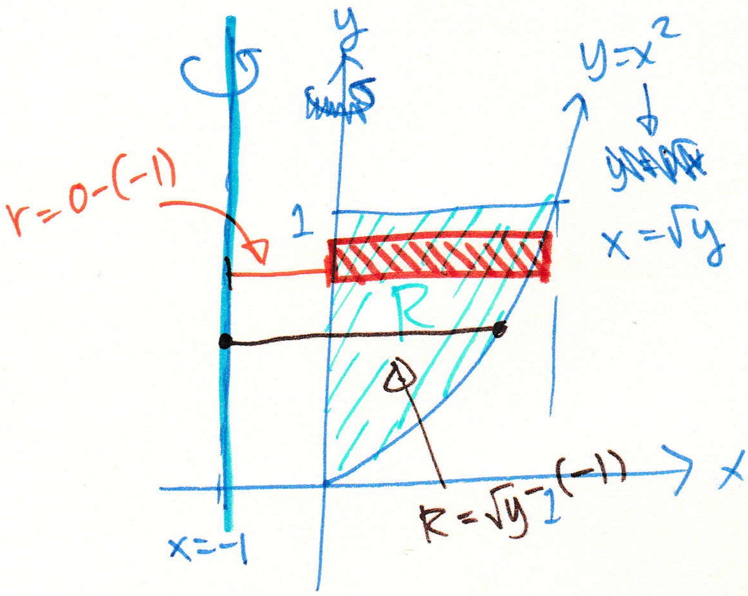
It is possible to use the method of disks and the method of washers to find the volume of a solid of revolution whose axis of revolution is a line other than one of the coordinate axes. Instead of developing a new formula for each situation, we will appeal to Formulas (6.3) and (6.4) and integrate an appropriate cross-sectional area to find the volume.

Example 6.10 Find the volume of the solid that is obtained when the region between the curve $y = x + 1$ and $y = 0$ over the interval $[0, 2]$ is rotated about the line $y = -1$.



$$V = \pi \int_0^2 \left[\left((x+1)-(-1)\right)^2 - \left(0-(-1)\right)^2\right] dx$$

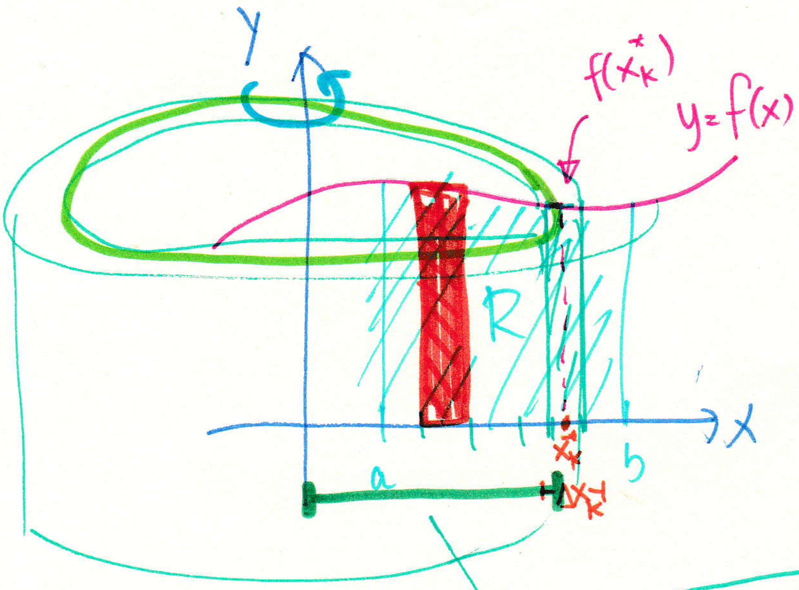
$$\boxed{V = \pi \int_0^2 \left[(x+2)^2 - (1)^2\right] dx} \quad \left(\frac{50\pi}{3}\right)$$

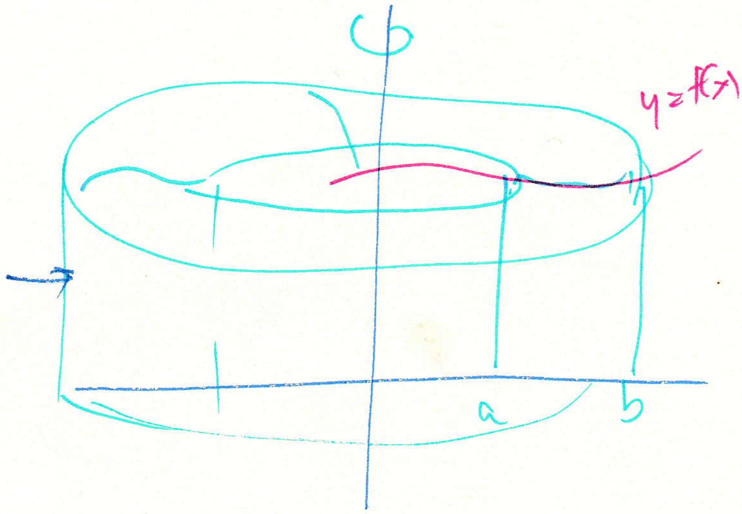


หมุนแบบ Washer รอบ $x = -1$

$$V = \int_0^1 \pi \left[\left(\sqrt{y}-(-1)\right)^2 - \left(0-(-1)\right)^2\right] dy$$

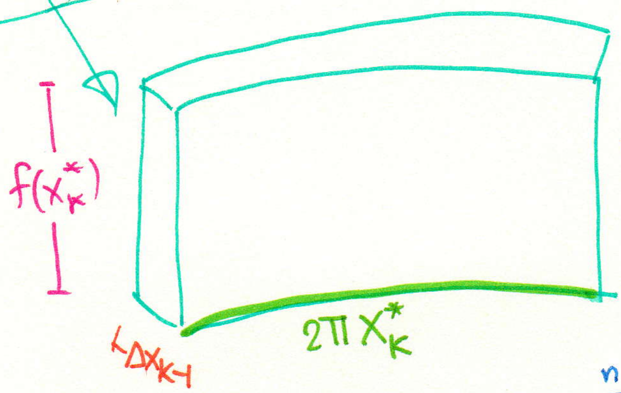
$$\boxed{V = \pi \int_0^1 \left[(\sqrt{y}+1)^2 - 1^2\right] dy}$$





ความยาวเส้นรอบวง

$2\pi x_k^*$



$$V_k = 2\pi x_k^* f(x_k^*)\Delta x_k$$

$$\therefore V = \lim_{n\to\infty}\sum_{k=1}^{n} 2\pi x_k^* f(x_k^*)\Delta x_k$$

$$\boxed{V = \int_a^b \underbrace{2\pi x}_{\text{ความยาวเส้นรอบวง}}\ \underbrace{f(x)}_{\text{ความสูง}}\ \underbrace{dx}_{\text{ความกว้าง}}}$$

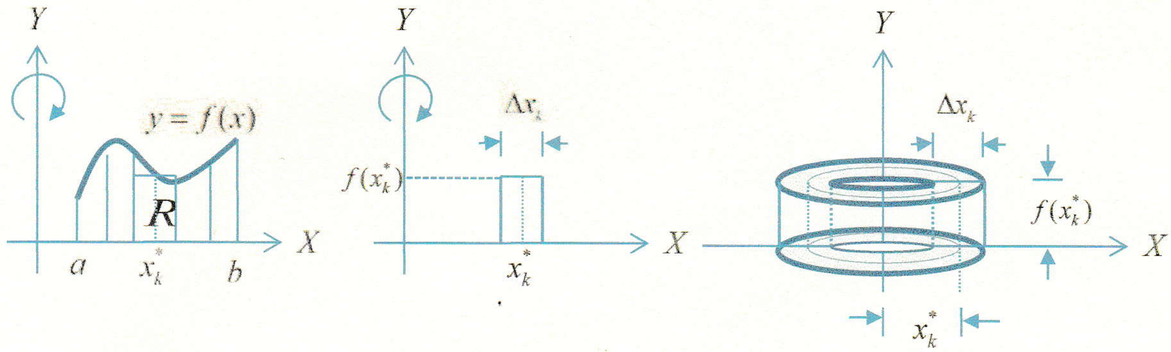
Concept

สร้างแถบให้ **ขนาน** กับแกนหมุน

6.3 Volumes by Cylindrical Shells การหมุนแบบเปลือกทรงกระบอก

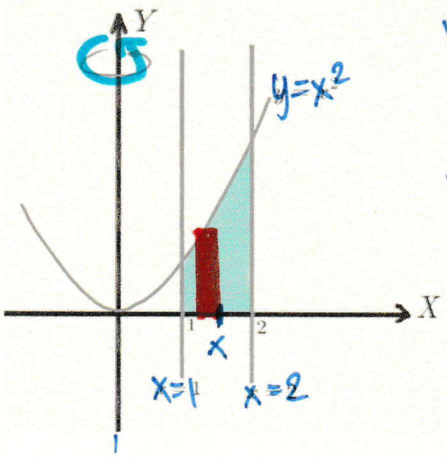
Theorem 6.5 (**Volume by cylindrical shells about the Y-axis**) Let f be continuous and nonnegative on [a, b] and let R be the region that is bounded above by $y = f(x)$, below by the X-axis, and on the sides by the lines $x = a$ and $x = b$. Then the volume V of the solid of revolution that is generated by revolving the region R about the Y-axis is given by

$$V = \int_a^b 2\pi x f(x)dx. \tag{6.9}$$



$$V = \lim_{max\Delta x_k \to 0} \sum_{k=1}^{n} 2\pi x_k^* f(x_k^*)\Delta x_k = \int_a^b 2\pi x f(x)dx.$$

Example 6.11 Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^2$, $x = 1$, $x = 2$ and the X-axis is revolved about the Y-axis.



$V = \int_1^2 2\pi x(x^2)\ dx$

$V = 2\pi \int_1^2 x^3 dx$

$= 2\pi \left[\frac{x^4}{4}\right]_1^2$

$= \frac{2\pi}{4}[16 - 1] = \frac{30\pi}{4} = \frac{15\pi}{2}$ #

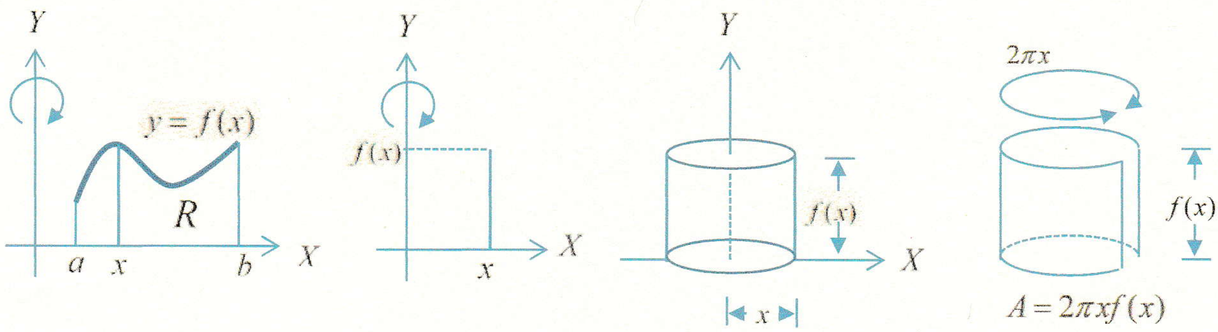
Variations of the method of cylindrical shells

The method of cylindrical shells is applicable in a variety of situations that do not fit the conditions required by Formula (6.9). For example, the region may be enclosed between two curves, or the axis of revolution may be some line other than the Y-axis. However, rather than develop a separate formula for every possible situation, we will give a general way of thinking about the method of cylindrical shells that can be adapted to each new situation as it arises. For this purpose, we will need to reexamine the integrand in Formula (6.9): At each x in the interval [a, b], the vertical line segment from the X-axis to the curve $y = f(x)$ can be viewed as the cross section of the region R at x. When the region R is revolved about the Y-axis, the cross section at x sweeps out the surface of a right circular cylinder of **height** $f(x)$ and **radius** x. The area of this surface is

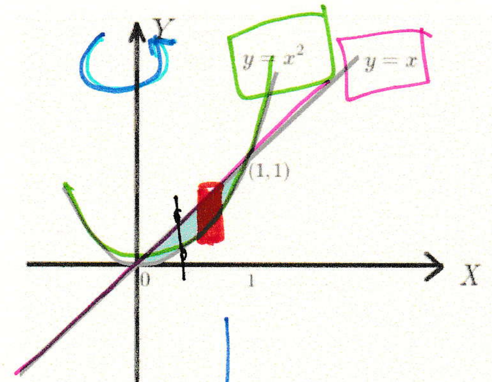
$$2\pi x f(x),$$

which is the integrand in (6.9). Thus, Formula (6.9) can be viewed informally in the following way.

An informal viewpoint about cylindrical shells: The volume V of a solid of revolution that is generated by revolving a region R about an axis can be obtained by integrating the area of the surface generated by an arbitrary cross section of R taken parallel to the axis of revolution.



Example 6.12 Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the Y-axis.



Cylindrical shell รอบแกน Y

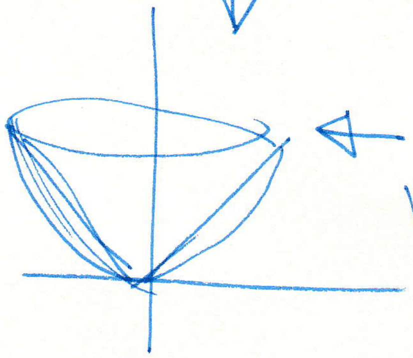
$$\boxed{V = \int_0^1 2\pi x \,[x - x^2]\, dx}$$

รัศมี ความสูง

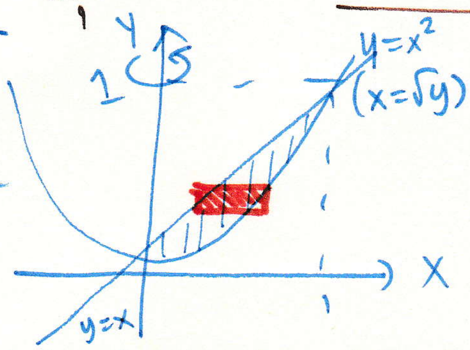
$$= 2\pi \int_0^1 (x^2 - x^3)\, dx = 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 2\pi\left[\frac{1}{3} - \frac{1}{4}\right] = 2\pi\left(\frac{4-3}{12}\right)$$

$$= \frac{\pi}{6} \quad \#$$



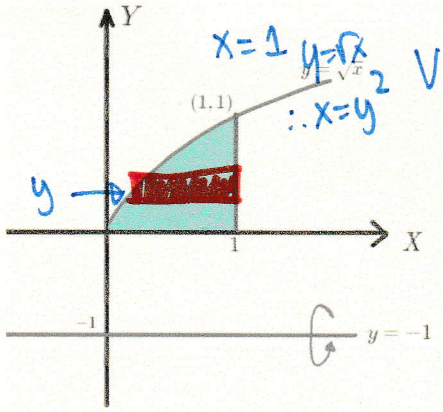
หาแบบ washer รอบแกน Y



$$\boxed{V = \int_0^1 \pi\left((\sqrt{y})^2 - y^2\right) dy}$$

$$= \int_0^1 \pi (y - y^2)\, dy \to \frac{\pi}{6}$$

Example 6.13 Use cylindrical shells to find the volume of the solid generated when the region R under $y = \sqrt{x}$ over the interval $[0, 1]$ is revolved about the line $y = -1$.



$x = 1$ $y = \sqrt{x}$ $\therefore x = y^2$

$$V = \int_0^1 2\pi(y - (-1))[1 - y^2]\, dy$$

$$\boxed{V = 2\pi \int_0^1 (y+1)(1-y^2)\, dy}$$

⋮

$$\boxed{V = \frac{11\pi}{6}}$$