เทคนิคการอินทิเกรต

① การอินทิเกรตแบบแบ่งส่วน (Integration by parts)

Ex $\int x\sin x\,dx$

$\int x\sin(x^2)\,dx$
$u = x^2$; $du = 2x\,dx$

$d(uv) = u\,dv + v\,du$

$\int d(uv) = \int u\,dv + \int v\,du$

$uv = \int u\,dv + \int v\,du$

$\therefore \boxed{\int u\,dv = uv - \int v\,du}$

เทคนิค ① เลือก u, dv ให้เหมาะสม
└ u diff ง่ายๆ
dv integrate ง่ายๆ

Ex 7.1 $\int x\sin x\,dx$

$u = x$ $\quad dv = \sin x\,dx$
$du = dx$ $\quad v = \int \sin x\,dx = -\cos x$

$\therefore \int x\sin x\,dx = (x)(-\cos x) - \int(-\cos x)\,dx$
$= -x\cos x + \sin x + C$ #

$\int x\sin x\,dx$

① $u = 1$; $dv = x\sin x\,dx$
$du = 0$; $v = \int x\sin x\,dx$ ☺

② $u = x$; $dv = \sin x\,dx$

③ $u = \sin x$; $dv = x\,dx$
$du = \cos x\,dx$; $v = \frac{x^2}{2}$
$\Big\} \int x\sin x\,dx = \frac{x^2}{2}\sin x - \int \frac{x^2}{2}\cos x\,dx$

④ $u = x\sin x$; $dv = dx$
$du = (x\cos x + \sin x)\,dx$; $v = x$
$\Big\} \int x\sin x\,dx = x^2\sin x - \int x(x\cos x + \sin x)\,dx$

Ex $\int \ln x \, dx$

(integration by parts)

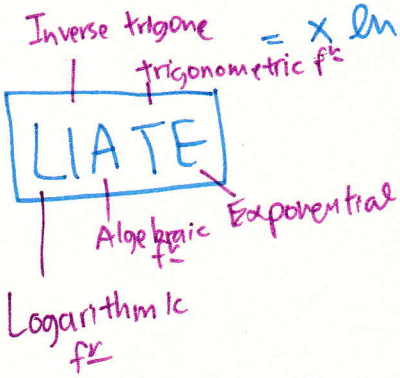
$u = \ln x$; $dv = dx$

$\therefore du = \frac{1}{x} dx$; $v = x$

$$\therefore \int \ln x \, dx = x \ln x - \int x\left(\frac{1}{x}\right) dx$$

$$= x \ln x - \int dx$$

$$= x \ln x - x + C \quad \#$$



Ex $\int e^x$

Example 7.2 Evaluate $\int x^3 \ln x \, dx$.

$u = \ln x \quad ; \quad dv = x^3 \, dx$

$du = \frac{1}{x} dx \quad ; \quad v = \frac{x^4}{4}$

$$\therefore \int x^3 \sin x \, dx = (\ln x)\left(\frac{x^4}{4}\right) - \int \left(\frac{x^4}{4}\right)\left(\frac{1}{x}\right) dx$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4}\int x^3 \, dx$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4}\left(\frac{x^4}{4}\right) + C \quad \#$$

7.2.2 Repeated integration by parts

Example 7.3 Evaluate $\int x^2 e^x \, dx$.

$u = x^2 \quad ; \quad dv = e^x \, dx$

$du = 2x \, dx \quad ; \quad v = e^x$

$$\therefore \int x^2 e^x \, dx = x^2 e^x - \int 2x e^x \, dx$$

$$= x^2 e^x - 2\boxed{\int x e^x \, dx}$$

ทรหา integrate by parts ตัวที่เหลือ ให้เลือกฟังก์ชันตัวเดิม.

$u_1 = x \quad ; \quad dv_1 = e^x \, dx$

$\therefore du_1 = dx \quad ; \quad v_1 = e^x$

$$\therefore \int x e^x \, dx = x e^x - \int e^x \, dx$$

$$= x e^x - e^x$$

$$\therefore \int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2e^x + C \quad \#$$

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Example 7.4 Evaluate $\int e^x \cos x \, dx$.

$u = \cos x \quad ; dv = e^x dx$

$\therefore du = -\sin x \, dx ; \quad v = e^x$

$\therefore \int e^x \cos x \, dx = e^x \cos x + \int -e^x \sin x \, dx$

$= e^x \cos x + \int e^x \sin x \, dx$

$u_1 = \sin x ; \quad dv_1 = e^x dx$

$du_1 = \cos x \, dx ; \quad v_1 = e^x$

$\therefore \int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$

$\therefore \int e^x \cos x \, dx = e^x \cos x + (e^x \sin x - \int e^x \cos x \, dx)$

$2\int e^x \cos x \, dx = e^x \cos x + e^x \sin x$

$\therefore \int e^x \cos x \, dx = \frac{e^x \cos x}{2} + \frac{e^x \sin x}{2} + C \quad \#$

7.2.3 Integration by parts for definite integrals

Example 7.5 Evaluate $\int_0^1 \tan^{-1} x \, dx$.

$u = \arctan x \quad ; \quad dv = dx$

$du = \frac{1}{1+x^2} dx ; \quad v = x.$

$\therefore \int_0^1 \arctan x \, dx = x \arctan x \Big|_0^1 - \int_0^1 \frac{x}{1+x^2} dx$

ให้ $t = 1+x^2 ; \quad dt = 2x \, dx \Rightarrow dx = \frac{1}{2x} dt$

$x = 0 \to t = 1$

$x = 1 \to t = 2$

$\therefore \int_0^1 \frac{x}{1+x^2} dx = \int_1^2 \frac{x}{t} \left(\frac{1}{2x}\right) dt$

$= \frac{1}{2} \int_1^2 \frac{1}{t} dt$

$= \frac{1}{2} \ln t \Big|_1^2$

$= x \arctan x \Big|_0^1 - \frac{1}{2} \ln t \Big|_1^2$

$= [\arctan 1 - 0] - [\frac{1}{2} \ln 2 - \frac{1}{2} \ln 1]$ ($\ln 1 \to 0$)

$= \pi/4 - \frac{1}{2} \ln 2 \quad \#$