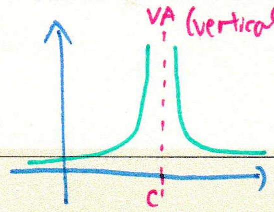
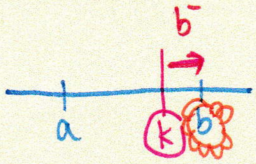
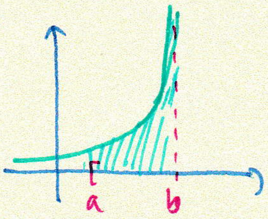


Definition If f is continuous on the interval $[a, b]$, except for an infinite discontinuity at b, then the improper integral of f over the interval $[a, b]$ is defined as

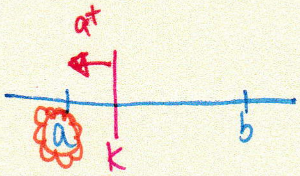


$$\int_a^b f(x)dx = \boxed{\lim_{k\to b^-}} \int_a^k f(x)dx,$$

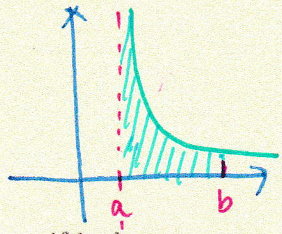


The integral is said to converge if the indicated limit exists and diverge if it does not.

If f is continuous on the interval $[a, b]$, except for an infinite discontinuity at a, then the improper integral of f over the interval $[a, b]$ is defined as



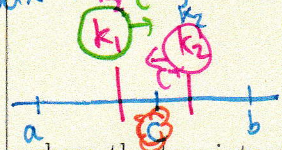
$$\int_a^b f(x)dx = \boxed{\lim_{k\to a^+}} \int_k^b f(x)dx,$$



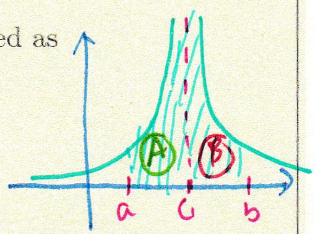
The integral is said to converge if the indicated limit exists and diverge if it does not.

If f is continuous on the interval $[a, b]$, except for an infinite discontinuity at a point c in (a, b), then the improper integral of f over the interval $[a, b]$ is defined as

$$\lim_{k_1\to c^-} \int_a^{k_1} f(x)dx + \lim_{k_2\to c^+} \int_{k_2}^{b} f(x)dx$$

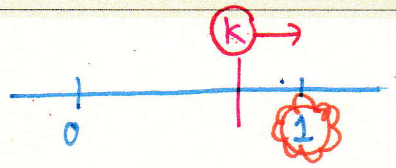


$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx,$$



where the two integrals on the right side are themselves improper. The improper integral on the left side is said to converge if both terms on the right side converge and diverge if either term on the right side diverges.

Example 7.26 Compute $\int_0^1 \frac{1}{1-x}\,dx$. มีปัญหาที่ $x = 1$



$$\int_0^1 \frac{1}{1-x}dx = \lim_{k\to 1^-} \int_0^k \frac{1}{1-x}dx$$

$u = 1-x$

$du = -dx$

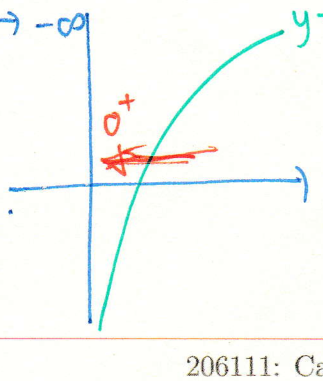
$$\int \frac{1}{1-x}dx = -\int \frac{1}{u}du$$
$$= -\ln|u| + C$$
$$= -\ln|1-x|$$

$$= \lim_{k\to 1^-} -\ln|1-x|\Big|_0^k$$

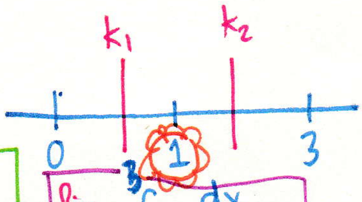
$$= \lim_{k\to 1^-} -\ln|1-k| + \ln|1-0|$$

ถ้า $k \to 1^-$ ได้ว่า $1-k \to 0^+$ ได้ว่า $\ln|1-k| \to -\infty$

$= +\infty$ diverges.



Example 7.27 Compute $\displaystyle\int_0^3 \frac{dx}{(x-1)^{2/3}}$.



$$\int_0^3 \frac{dx}{(x-1)^{2/3}} = \boxed{\lim_{k_1\to1^-}\int_0^{k_1}\frac{dx}{(x-1)^{2/3}}} + \boxed{\lim_{k_2\to1^+}\int_{k_2}^{3}\frac{dx}{(x-1)^{2/3}}}$$

หา

$$\int \frac{dx}{(x-1)^{2/3}} \quad u=x-1;\ du=dx$$

$$= \int \frac{du}{u^{2/3}} = \frac{3u^{1/3}}{1}+C = 3(x-1)^{1/3}+C.$$

พิจารณา $\displaystyle\lim_{k_1\to1^-}\int_0^{k_1}\frac{dx}{(x-1)^{2/3}} = \lim_{k_1\to1^-} 3(x-1)^{1/3}\Big|_0^{k_1} = \lim_{k_1\to1^-}\big[3(k_1-1)^{1/3} - \underbrace{3(0-1)^{1/3}}_{3}\big] = 3$

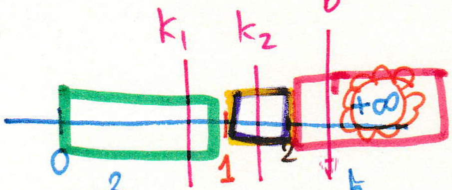
พิจารณา $\displaystyle\lim_{k_2\to1^+}\int_{k_2}^{3}\frac{dx}{(x-1)^{2/3}} = \lim_{k_2\to1^+} 3(x-1)^{1/3}\Big|_{k_2}^{3} = \lim_{k_2\to1^+}\big[3(3-1)^{1/3} - 3(k_2-1)^{1/3}\big] = 3\sqrt[3]{2}$

$\therefore \displaystyle\int_0^3 \frac{dx}{(x-1)^{2/3}} = 3+3\sqrt[3]{2}$ converges.

Type 3

The following examples are improper integrals with infinite discontinuities over infinite intervals of integration. Remark that it combines the first and second type of improper integral. With this type, we should separate the improper integral into the first and second type. Then, the convergence will be considered using the mentioned methods.

Example 7.28 Compute $\displaystyle\int_0^\infty \frac{dx}{(x-1)^{2/3}}$.



$$\int \frac{dx}{(x-1)^{2/3}} = 3(x-1)^{1/3}+C$$

$$\therefore \int_0^{+\infty}\frac{dx}{(x-1)^{2/3}} = \underline{\lim_{k_1\to1^-}\int_0^{k_1}\frac{dx}{(x-1)^{2/3}}} + \underline{\lim_{k_2\to1^+}\int_{k_2}^{2}\frac{dx}{(x-1)^{2/3}}} + \underline{\lim_{b\to+\infty}\int_2^{b}\frac{dx}{(x-1)^{2/3}}}$$

$x=1$

พิจารณา $\displaystyle\lim_{k_1\to1^-}\int_0^{k_1}\frac{dx}{(x-1)^{2/3}} = \lim_{k_1\to1^-}\big[3(x-1)^{1/3}\big]_0^{k_1} = 3$

พิจารณา $\displaystyle\lim_{k_2\to1^+}\int_{k_2}^{2}\frac{dx}{(x-1)^{2/3}} = \lim_{k_2\to1^+}\big[3(2-1)^{1/3} - 3(k_2-1)^{1/3}\big] = 3$

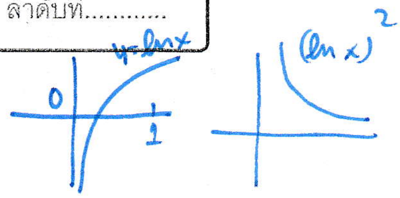
พิจารณา $\displaystyle\lim_{b\to\infty}\int_2^{b}\frac{dx}{(x-1)^{2/3}} = \lim_{b\to\infty}\big[3(b-1)^{1/3} - 3(2-1)^{1/3}\big] = +\infty \rightarrow \int_0^{+\infty}\frac{dx}{(x-1)^{2/3}}$ diverges!

Example 7.29 Compute $\displaystyle\int_{-\infty}^{0} \frac{e^{1/x}}{x^2}dx$.

แบบทดสอบย่อย

เพื่อเช็คชื่อเข้าชั้นเรียน ประจำวันศุกร์ที่ 19 เมษายน พ.ศ.2562

ชื่อ-สกุล.. รหัสนักศึกษา.......................... ลำดับที่...........



1. จงเขียนอินทิกรัลไม่ตรงแบบต่อไปนี้ในรูปลิมิตโดยไม่ต้องคำนวณค่าลิมิต

(a) $\int_0^1 (\ln x)^2\, dx$ Type 2 $= \lim_{k\to 0^+} \int_k^1 (\ln x)^2\, dx$

(b) $\int_0^{+\infty} \frac{1}{(x-1)^2}\, dx$ Type3 $x=1$ = ...

(c) $\int_1^{+\infty} \frac{1}{x(x-2)}\, dx$ Type 3 $x=2$ = ...

(d) $\int_{-\infty}^{+\infty} \frac{1}{x^2+3}\, dx$ $= \boxed{\int_{-\infty}^{0} \frac{1}{x^2+3}\, dx} + \boxed{\int_{0}^{+\infty} \frac{1}{x^2+3}\, dx} = \lim_{a\to -\infty} \int_a^0 \frac{1}{x^2+3}\, dx + \lim_{b\to +\infty} \int_0^b \frac{1}{x^2+3}\, dx$

(e) $\int_1^{+\infty} x\, dx$ $= \lim_{b\to +\infty} \int_1^b x\, dx$

(f) $\int_{-\infty}^{2} \frac{1}{(x-2)^4}\, dx$ Type 3 $x=2$ = ...

(g) $\int_1^2 \frac{x}{(x^2-1)}\, dx$ Type 2 $x=1$ = ...

2. จงพิจารณาว่าอินทิกรัล $\int_{-\infty}^{-1} \frac{1}{x^5}\, dx$ ลู่เข้าหรือลู่ออก และถ้าลู่เข้า จงคำนวณค่าของอินทิกรัล

3. จงแสดงวิธีการหาค่าปริพันธ์ไม่ตรงแบบ $\int_1^2 \frac{x}{(x^2-1)}\, dx$ พร้อมทั้งระบุว่าปริพันธ์ดังกล่าวลู่เข้าหรือลู่ออก