$\boxed{\text{Ex 3.13}}$

(Implicit Function)

$$x^3 + x\arctan y = y \quad \text{หา } \frac{dy}{dx}$$

$$\frac{d}{dx}x^3 + \frac{d}{dx}(x\arctan y) = \frac{dy}{dx}$$

$$3x^2 + x\frac{d}{dx}\arctan y + \arctan y \cancelto{1}{\frac{dx}{dx}} = \frac{dy}{dx}$$

$$3x^2 + x\left(\frac{1}{1+y^2}\frac{dy}{dx}\right) + \arctan y = \frac{dy}{dx}$$

$$\frac{x}{1+y^2}\frac{dy}{dx} - \frac{dy}{dx} = -3x^2 - \arctan y$$

$$\left(\frac{x}{1+y^2} - 1\right)\frac{dy}{dx} = -3x^2 - \arctan y$$

$$\frac{dy}{dx} = \frac{-3x^2 - \arctan y}{\frac{x}{1+y^2} - 1}$$

#

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

ให้ $y = \arcsin x$ หา $\frac{dy}{dx}$

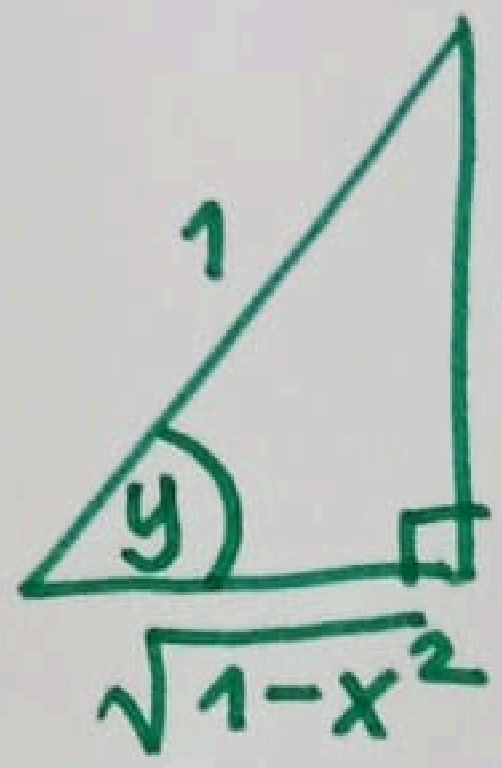
$$\sin y = x$$

$$\frac{d}{dx}\sin y = \frac{dx}{dx}$$

$$\cos y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$= \frac{1}{\sqrt{1-x^2}}$$



$$\frac{d}{dx}\arcsin u = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

Ex

$$\frac{d}{dx}\arcsin(x^3) = \frac{1}{\sqrt{1-x^6}}\frac{d}{dx}x^3$$

$$= \frac{1}{\sqrt{1-x^6}}(3x^2)$$

$$\frac{d}{dx}\arcsin(e^x) = \frac{1}{\sqrt{1-e^{2x}}}(e^x)$$

$$\frac{d}{dx}2^x\arcsin x = 2^x\left(\frac{1}{\sqrt{1-x^2}}\right) + \arcsin x\,(2^x\ln 2)$$

Ex 3.11

$$y = \arcsin(x^3+1)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(x^3+1)^2}}\frac{d}{dx}(x^3+1)$$

$$= \frac{3x^2}{\sqrt{1-(x^3+1)^2}}$$

~~Ex 3.12~~

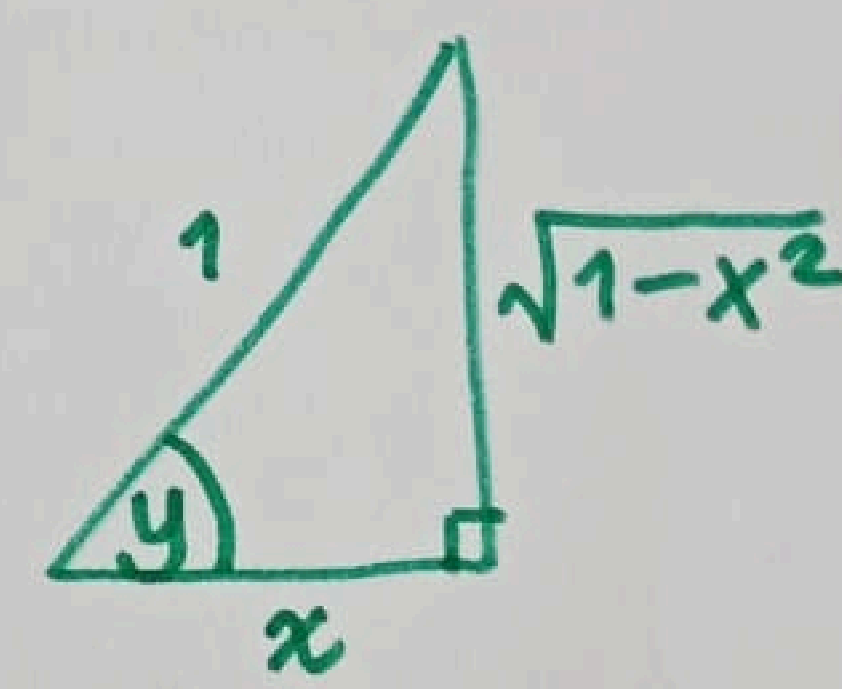
$$\frac{d}{dx}\arccos x = \ldots$$

ให้ $y = \arccos x$ หา $\frac{dy}{dx}$

$$\cos y = x$$

$$\frac{d}{dx}\cos y = \frac{dx}{dx}$$

$$-\sin y \frac{dy}{dx} = 1$$



$$\frac{dy}{dx} = -\frac{1}{\sin y}$$

$$= \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arccos u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$$

Ex

$$\frac{d}{dx}\arccos(x^3) = \frac{-1}{\sqrt{1-x^6}}\frac{d}{dx}x^3$$

$$= \frac{-3x^2}{\sqrt{1-x^6}}$$

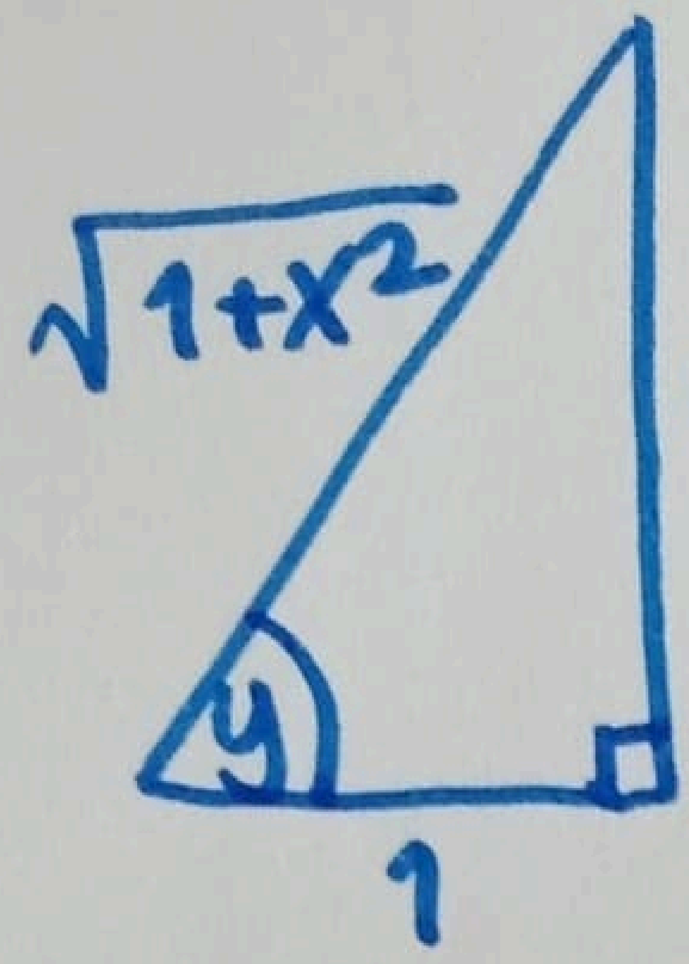
Ex 3.12

$$y = \arccos\sqrt{x}$$

$$y' = \frac{-1}{\sqrt{1-(\sqrt{x})^2}}\frac{d}{dx}\sqrt{x}$$

$$= \frac{-1}{\sqrt{1-x}}\left(\frac{1}{2\sqrt{x}}\right)$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$



$\sec y = \frac{\sqrt{1+x^2}}{1}$

ให้ $y = \arctan x$, หา $\frac{dy}{dx}$

$$\tan y = x$$

$$\frac{d}{dx}\tan y = \frac{dx}{dx}$$

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\arctan u = \frac{1}{1+u^2}\frac{du}{dx}$$

Ex

$$\frac{d}{dx}\arctan(3^x) = \frac{1}{1+(3^x)^2}\frac{d}{dx}3^x$$

$$= \frac{1}{1+3^{2x}}(3^x \ln 3)$$

3.5 Calculus as a Rate of Change อัตราการเปลี่ยนแปลง

In scientific works, we always deal with change of phenomena such as the number of population, the concentration of reactant after chemical reaction, or rectilinear motion. In this section, we consider an example of the rectilinear motion as a rate of change.

Rectilinear Motions (การเคลื่อนที่เป็นเส้นตรง)

The position coordinate of a particle in rectilinear motion at time t is $s = f(t)$. The *average velocity* of the particle over a time interval $[t_0, t_0 + h]$ for $h > 0$ is

ความเร็วเฉลี่ย

$$v_{ave} = \frac{\text{change in position}}{\text{time elapsed}} = \frac{f(t_0 + h) - f(t_0)}{h}.$$

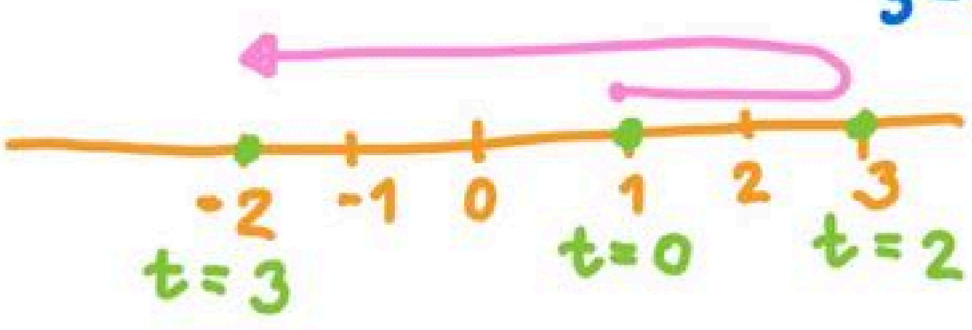
Example 3.17 Suppose that $s = f(t) = 1 + 5t - 2t^2$ is the position function of a particle, where s is in meters and t is in seconds. Find the average velocities of the particle over the time intervals (a) $[0, 2]$ and (b) $[2, 3]$.

Solution. (a) ช่วงเวลา $[0,2]$

$$v_{ave} = \frac{s(2) - s(0)}{2 - 0} = \frac{3-1}{2} = 1 \text{ m/s}$$

(b) ช่วงเวลา $[2,3]$

$$v_{ave} = \frac{s(3) - s(2)}{3 - 2} = \frac{-2-3}{1} = -5 \text{ m/s}$$



3.5 Rate of Change $\qquad y = f(x)$

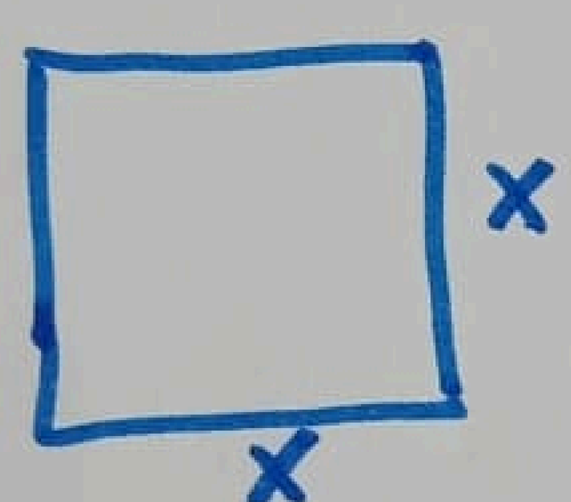
อัตราการเปลี่ยนแปลง

อนุพันธ์

→ อัตราการเปลี่ยนแปลงเฉลี่ย $\qquad \dfrac{f(x) - f(x_0)}{x - x_0}$

→ อัตราการเปลี่ยนแปลง ณ ขณะใดขณะหนึ่ง

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$



x = ความยาวด้าน

A = พื้นที่

$$A(x) = x^2$$

$$A'(x) = 2x$$

$x = 10$ cm;

$A = 100\ \text{cm}^2$

$$A'(10) = 2(10) = 20\ \text{cm}^2/\text{cm}$$

$x = 11$ cm

$A = 121\ \text{cm}^2$

อัตรา~~...~~การเปลี่ยนแปลงเฉลี่ย $= \dfrac{A(11) - A(10)}{11 - 10}$

$$= \frac{121 - 100}{1}$$

$$= 21\ \text{cm}^2/\text{cm}$$