Integration derivative อนุพันธ์

→ Antiderivative (ปฏิยานุพันธ์)

$F(x)$ เป็น antiderivative ของ $f(x)$ $\Leftrightarrow$ $F'(x) = f(x)$

Ex $f(x) = x^2$

$$\left.\begin{aligned} F(x) &= \frac{x^3}{3} \\ F(x) &= \frac{x^3}{3} + 1 \\ F(x) &= \frac{x^3}{3} - 2 \\ &\vdots \end{aligned}\right\} \quad F'(x) = f(x)$$

$$F(x) + C$$

(indefinite) Integral

$$\int f(x)\,dx = F(x) + C, \quad F'(x) = f(x)$$

integrand

variable (ตัวแปร)

integration - การอินทิเกรต

Ex $\int \cos x\,dx = \sin x + C$

$\int e^x\,dx = e^x + C$

สูตรพื้นฐาน

$$\frac{d}{dx}x^n = nx^{n-1} \quad , \quad \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\frac{d}{dx}e^x = e^x \qquad \int e^x dx = e^x + C$$

$$\frac{d}{dx}a^x = a^x \ln a \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\frac{d}{dx}\sin x = \cos x \qquad \int \cos x dx = \sin x + C$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \int \sin x dx = -\cos x + C$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \int \sec^2 x dx = \tan x + C$$

$$\int \operatorname{cosec}^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \qquad \int \frac{1}{1+x^2} dx = \arctan x + C$$

Ex 5.1

(a) $\int (x^6 - 7x + 4)\,dx = \frac{x^7}{7} - 7\frac{x^2}{2} + 4x + C$

$\int t^4 + 3t^2\,dt = \frac{t^5}{5} + 3\frac{t^3}{3} + C$

$\int (u^3 - 1)\,du = \frac{u^4}{4} - u + C$

(b) $\int \frac{x^5 + 2x^3 - 1}{x^4}\,dx \neq \frac{\int x^5 + 2x^3 - 1\,dx}{\int x^4\,dx}$

$\int \left(x + \frac{2}{x} - \frac{1}{x^4}\right)dx = \frac{x^2}{2} + 2\ln|x| - \frac{x^{-3}}{-3} + C$

(c) $\int \sqrt{x} + \frac{1}{\sqrt[3]{x}}\,dx = \int x^{1/2} + x^{-1/3}\,dx$

$= \frac{x^{3/2}}{3/2} + \frac{x^{2/3}}{2/3} + C$

$= \frac{2}{3}x^{3/2} + \frac{3}{2}x^{2/3} + C$

(d) $\int e^x + 2^x\,dx = e^x + \frac{2^x}{\ln 2} + C$

(e) $\int 4\sin x + 2\cos x\,dx = -4\cos x + 2\sin x + C$

(f) $\int \frac{3}{\sqrt{1-x^2}} - \frac{2}{1+x^2}\,dx = 3\arcsin x - 2\arctan x + C$

Integration by substitution
(การอินทิเกรตโดยการเปลี่ยนตัวแปร)

$$\int f(x)\,dx \rightsquigarrow \int F(u)\,du$$

Ex

$$\int \cos x\,dx = \sin x + C$$

$$\int \cos(\underbrace{5x+1}_{u})\,dx$$

ให้ $u = 5x+1$

$du = 5dx$

$\frac{du}{5} = dx$

$$= \int \cos u \frac{du}{5}$$

$$= \frac{1}{5}\sin u + C$$

$$= \frac{1}{5}\sin(5x+1) + C$$

Ex 5.2

$$\int (2x+1)^{100}\,dx$$

ให้ $u = 2x+1$

$du = 2dx$

$$= \int u^{100} \frac{du}{2}$$

$$= \frac{1}{2}\left(\frac{u^{101}}{101}\right) + C$$

$$= \frac{1}{2}\left[\frac{(2x+1)^{101}}{101}\right] + C$$

Ex 5.3

$$\int \frac{x^2}{(3x^3-2)^9}\,dx$$

$$= \int \frac{1}{u^9}\left(\frac{du}{9}\right)$$

$$= \frac{1}{9}\left(\frac{u^{-8}}{-8}\right) + C$$

$$= \frac{-1}{72}(3x^3-2)^{-8} + C$$

ให้ $u = 3x^3 - 2$

$du = 9x^2\,dx$

$\frac{du}{9} = x^2\,dx$

Ex 5.4

$$\int \cos(5x)\,dx$$

$$= \int \cos u \,\frac{du}{5}$$

$$= \frac{1}{5}\sin u + C$$

$$= \frac{1}{5}\sin(5x) + C$$

ให้ $u = 5x$

$du = 5\,dx$

$\frac{du}{5} = dx$

Ex 5.5

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$$

$$= \int \cos u\,(2\,du)$$

$$= 2\sin u + C$$

$$= 2\sin\sqrt{x} + C$$

ให้ $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

(Ex5.6) $\int e^x \sec^2(e^x+1)\,dx$

Let $u = e^x + 1$

$du = e^x dx$

$= \int \sec^2 u \, du$

$= \tan u + C$

$= \tan(e^x+1) + C$