ตัวอย่าง

Ex $\int 3x(2x-7)\,dx = \int (6x^2 - 21x)\,dx$

$= 6\cdot\frac{x^3}{3} - 21\frac{x^2}{2} + C$

$= 2x^3 - \frac{21}{2}x^2 + C$ #

Quiz 1 $\int \left(\frac{1}{6\sqrt{1-x^2}} - \frac{7}{1+x^2}\right)dx = \frac{1}{6}\int \frac{1}{\sqrt{1-x^2}}\,dx - 7\int \frac{1}{1+x^2}\,dx$

($\frac{1}{6\sqrt{1-x^2}}$: $\arcsin x$, $\frac{7}{1+x^2}$: $\arctan x$)

$= \frac{1}{6}\arcsin x - 7\arctan x + C$ #

Quiz 2 $\int \cos(\underbrace{9x+1}_{u})\underbrace{dx}_{du} \to \int \cos u\,du$

เลือก $u = 9x+1$

$du = 9dx$

$dx = \frac{1}{9}du$

$\therefore \int \cos(9x+1)\,dx = \int \cos u \cdot \frac{1}{9}\,du$

$= \frac{1}{9}\int \cos u\,du$

$= \frac{1}{9}\sin u + C$

$= \frac{1}{9}\sin(9x+1) + C$ #

Quiz 3 $\int \frac{\sec^2(\underbrace{\ln x}_{u})}{x}\underbrace{dx}_{du}$

$u = \ln x$

$\therefore du = \frac{1}{x}dx$

$\Rightarrow dx = x\,du$

(I) $\int \frac{\sec^2(\ln x)}{x}dx = \int \frac{\sec^2 u}{x}\cdot x\,du$

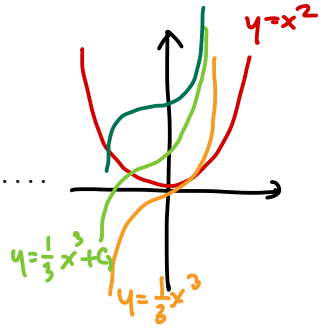
$= \int \sec^2 u\,du$

$= \tan u + C$

$= \tan(\ln x) + C$. #

(II) $\int \frac{\sec^2(\ln x)}{x}dx \xrightarrow{du} \int \sec^2 u\,du = \tan u + C = \tan(\ln x) + C$

For example, $\frac{1}{3}x^3$, $\frac{1}{3}x^3+1$, $\frac{1}{3}x^3-3$, $\frac{1}{3}x^3-\sqrt{2}$ are all antiderivatives of $f(x)=\ldots x^2\ldots\ldots$



5.1.2 Integration Formulas

Differentiation Formula	Integration Formula
1. $\frac{d}{dx}(C)=0$	1. $\int 0dx=C$
2. $\frac{d}{dx}[kx]=k$	2. $\int kdx=kx+C$
3. $\frac{d}{dx}[kf(x)]=kf'(x)$	3. $\int [kf(x)]dx=k\int f(x)dx$
4. $\frac{d}{dx}[f(x)\pm g(x)]=f'(x)\pm g'(x)$	4. $\int [f(x)\pm g(x)]dx=\int f(x)dx\pm\int g(x)dx$
5. $\frac{d}{dx}[x^n]=nx^{n-1}$	5. $\int x^n dx=\frac{x^{n+1}}{n+1}+C,\ n\neq -1$
6. $\frac{d}{dx}[\ln\lvert x\rvert]=\frac{1}{x}$	6. $\int \frac{1}{x}dx=\ln\lvert x\rvert+C$
7. $\frac{d}{dx}[e^x]=e^x$	7. $\int e^x dx=e^x+C$
8. $\frac{d}{dx}[a^x]=a^x\ln a,\ a>0$ and $a\neq 1$	8. $\int a^x dx=\frac{a^x}{\ln a}+C,\ a>0$ and $a\neq 1$
9. $\frac{d}{dx}[\sin x]=\cos x$	9. $\int \cos x dx=\sin x+C$
10. $\frac{d}{dx}[\cos x]=-\sin x$	10. $\int \sin x dx=-\cos x+C$
11. $\frac{d}{dx}[\tan x]=\sec^2 x$	11. $\int \sec^2 x dx=\tan x+C$
12. $\frac{d}{dx}[\cot x]=-\mathrm{cosec}^2 x$	12. $\int \mathrm{cosec}^2 x dx=-\cot x+C$
13. $\frac{d}{dx}[\sec x]=\sec x\tan x$	13. $\int \sec x\tan x dx=\sec x+C$
14. $\frac{d}{dx}[\mathrm{cosec}\, x]=-\mathrm{cosec}\, x\cot x$	14. $\int \mathrm{cosec}\, x\cot x dx=-\mathrm{cosec}\, x+C$
15. $\frac{d}{dx}[\arcsin x]=\frac{1}{\sqrt{1-x^2}}$	15. $\int \frac{1}{\sqrt{1-x^2}}dx=\arcsin x+C$
16. $\frac{d}{dx}[\arctan x]=\frac{1}{1+x^2}$	16. $\int \frac{1}{1+x^2}dx=\arctan x+C$
17. $\frac{d}{dx}[\ln\lvert\sec x\rvert]=\tan x$	17. $\int \tan x dx=\ln\lvert\sec x\rvert+C$
18. $\frac{d}{dx}[\ln\lvert\sin x\rvert]=\cot x$	18. $\int \cot x dx=\ln\lvert\sin x\rvert+C$

Example 5.5 Evaluate $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$

Example 5.6 Evaluate $\int e^x \sec^2(e^x + 1) dx$

Example 5.7 Evaluate $\int \frac{1}{\sqrt{2 - x^2}} dx$

$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$

$\int \frac{1}{\sqrt{2-x^2}} dx = \int \frac{1}{\sqrt{2(1-\frac{x^2}{2})}} dx = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{1-\frac{x^2}{2}}} dx$

$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{1-(\frac{x}{\sqrt{2}})^2}} dx$ (du)

ให้ $u = \frac{x}{\sqrt{2}}$; $du = \frac{1}{\sqrt{2}} dx$

$= \int \frac{1}{\sqrt{1-u^2}} du$

$= \arcsin u + C$

$= \arcsin\left(\frac{x}{\sqrt{2}}\right) + C$. #

⊛ $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right) + C$

$\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$

6

Techniques of Integration

6.1 Overview of Integration Methods

A review of familiar integration formulas

1. $\int du = u + C$
2. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
3. $\int \frac{1}{u}\, du = \ln|u| + C$
4. $\int a^u\, du = \frac{a^u}{\ln a} + C,\ a > 0, a \neq 1$
5. $\int e^u\, du = e^u + C$
6. $\int \sin u\, du = -\cos u + C$
7. $\int \cos u\, du = \sin u + C$
8. $\int \sec^2 u\, du = \tan u + C$
9. $\int \csc^2 u\, du = -\cot u + C$
10. $\int \sec u \tan u\, du = \sec u + C$
11. $\int \csc u \cot u\, du = -\csc u + C$
12. $\int \tan u\, du = ln|\sec u| + C$
13. $\int \cot u\, du = ln|\sin u| + C$
14. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin(\frac{u}{a}) + C$
15. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan(\frac{u}{a}) + C$

Ex $\int x \sin x \, dx = \int x\,dx \cdot \int \sin x \, dx$

สมมติ หัวกับ u คูณกับ v $\quad d(uv) = u\,dv + v\,du$

$$\int d(uv) = \int u\,dv + \int v\,du$$

$$uv = \int u\,dv + \int v\,du$$

$$\int u\,dv = uv - \int v\,du$$

$\int 1\,dx = x + C$

$\int 1\,dv = v + C$

$\int 1\,d\left(\frac{1}{\sqrt{x^2+1}}\right) = \frac{1}{\sqrt{x^2+1}} + C$

โจทย์ปัญหาอินทิเกรต ① integration by parts หรือเทคนิคการแยกส่วน

Ex $\int x \sin x \, dx$ เลือก u, dv $\quad \int u\,dv = uv - \int v\,du$

↓ หา du, v

เลือก u ให้ ดิฟง่ายๆ, เลือก dv ให้ อินทิเกรตง่ายๆ

เลือก $u = x$; $dv = \sin x\,dx$

$du = dx$; $v = \int \sin x\,dx$

$\therefore v = -\cos x$

$\therefore \int x \sin x \, dx = (x)(-\cos x) - \int (-\cos x)\,dx$

$= -x\cos x + \int \cos x \, dx$

$= -x\cos x + \sin x + C$ #

$\int x \sin x \, dx \rightarrow u = 1$; $dv = x\sin x\,dx$

$du = 0\,dx$; $v = \int x \sin x \, dx$

$u = \sin x$; $dv = x\,dx$

$du = \cos x\,dx$; $v = \frac{x^2}{2}$

$\therefore \int x \sin x \, dx = \frac{x^2}{2}\sin x - \int \frac{x^2}{2}\cos x\,dx$ [ไม่ดี]

$u = x\sin x$; $dv = dx$

$du = (x\cos x + \sin x)dx$; $v = x$

$\therefore \int x \sin x \, dx = x^2 \sin x - \int x \cdot (x\cos x + \sin x)\,dx$ (ไม่ดี)

6.2 Integration by Parts

6.2.1 The Product Rule vs Integration by Parts

Let $G(x)$ be any antiderivative of $g(x)$; $G'(x) = g(x)$

$$\frac{d}{dx}[f(x)G(x)] = f(x)G'(x) + f'(x)G(x) = f(x)g(x) + f'(x)G(x)$$

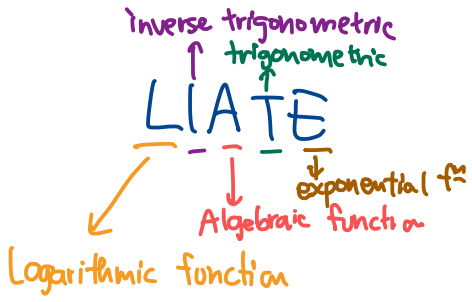
$$\int [f(x)g(x) + f'(x)G(x)]\ dx = f(x)G(x)$$

$$\int f(x)g(x)\ dx = f(x)G(x) - \int f'(x)G(x)\ dx$$

Let $u = f(x), du = f'(x)dx$ and $v = G(x), dv = g(x)\ dx$

$$\int udv = uv - \int vdu$$

Example 6.1 Evaluate $\int x \sin x\ dx$.



Example 6.2 Evaluate $\int x^3 \ln x \, dx$.

$u = \ln x$; $dv = x^3 dx$

$du = \frac{1}{x} dx$; $v = \frac{x^4}{4}$

$$\int x^3 \ln x \, dx = \ln x \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4}\int x^3 dx$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C \quad \#$$

6.2.2 Repeated Integration by Parts

Example 6.3 Evaluate $\int x^2 e^x \, dx$.

$u = x^2$; $dv = e^x dx$

$du = 2x dx$; $v = e^x$

$$\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx \quad \text{--- (*)}$$

สังเกตว่า integrate by part ของตัวนี้ จะใช้ u, du เป็นตัวเลข

พิจารณา $\int xe^x dx$ $u = x$; $dv = e^x$

$du = dx$; $v = e^x dx$

$$\therefore \int xe^x dx = xe^x - \int e^x dx$$

$$= xe^x - e^x$$

(*) $$\int x^2 e^x dx = x^2 e^x - 2(xe^x - e^x) + C$$

$$= x^2 e^x - 2xe^x + 2e^x + C \quad \#$$

Ex $\int \ln x \, dx$

$u = \ln x$; $dv = dx$

$\therefore du = \frac{1}{x} dx$; $v = x$

$\int u \, dv = uv - \int v \, du$

$\therefore \int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} dx$

$= x \ln x - \int dx$

$= x \ln x - x + C$ #

Example 6.4 Evaluate $\int e^x \cos x \, dx$.

Example 6.5 Evaluate $\int \arctan x \, dx$.

$u = \arctan x \quad ; \quad dv = dx$

$du = \frac{1}{1+x^2} dx \quad ; \quad v = x$

$\therefore \int \arctan x \, dx = x \arctan x - \int \frac{x}{1+x^2} dx$

วิชา 206111 Calculus 1

แบบฝึกหัดทบทวน: การเปลี่ยนตัวแปร (พื้นฐาน)...ฝึกฝน ฝึกฝน

สูตร 1 $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1} + C\,, \quad n \neq -1$

จงหาอินทิกรัลต่อไปนี้

$\displaystyle\int (2x+1)^3\,dx$ $\displaystyle\int \sqrt{2x+1}\,dx$ $\displaystyle\int \frac{1}{(2x+1)^2}\,dx$ $\displaystyle\int \frac{1}{(\sqrt{2x+1})}\,dx$

$\displaystyle\int x(3x^2-1)^3\,dx$ $\displaystyle\int x\sqrt{3x^2-1}\,dx$ $\displaystyle\int \frac{x}{(3x^2-1)^2}\,dx$ $\displaystyle\int \frac{x}{\sqrt{3x^2-1}}\,dx$

$\displaystyle\int \sin^3 x\cos x\,dx$ $\displaystyle\int \frac{\cos x}{\sqrt{\sin x}}\,dx$ $\displaystyle\int \sqrt{\sin x}\cos x\,dx$ $\displaystyle\int \frac{\cos x}{\sin^2 x}\,dx$

$\displaystyle\int \cos^3 x\sin x\,dx$ $\displaystyle\int \frac{\sin x}{\sqrt{\cos x}}\,dx$ $\displaystyle\int \sqrt{\cos x}\sin x\,dx$ $\displaystyle\int \frac{\sin x}{\cos^2 x}\,dx$

$\displaystyle\int \tan^3 x\sec^2 x\,dx$ $\displaystyle\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$ $\displaystyle\int \sqrt{\tan x}\sec^2 x\,dx$ $\displaystyle\int \frac{\sec^2 x}{\tan^2 x}\,dx$

$\displaystyle\int \cot^3 x\operatorname{cosec}^2 x\,dx$ $\displaystyle\int \frac{\operatorname{cosec}^2 x}{\sqrt{\cot x}}\,dx$ $\displaystyle\int \sqrt{\cot x}\operatorname{cosec}^2 x\,dx$ $\displaystyle\int \frac{\operatorname{cosec}^2 x}{\cot^2 x}\,dx$

$\displaystyle\int (e^x+1)^3 e^x\,dx$ $\displaystyle\int (5^x+1)^3 5^x\,dx$ $\displaystyle\int \frac{\arcsin^3 x}{\sqrt{1-x^2}}\,dx$ $\displaystyle\int \frac{\arctan^3 x}{1+x^2}\,dx$

สูตร 2 $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C$

จงหาอินทิกรัลต่อไปนี้

$\displaystyle\int \frac{1}{2x+1}\,dx$ $\displaystyle\int \frac{x}{x^2+1}\,dx$ $\displaystyle\int \frac{x^2}{x^3+1}\,dx$ $\displaystyle\int \frac{x^3}{x^4+1}\,dx$

$\displaystyle\int \frac{\cos x}{\sin x}\,dx$ $\displaystyle\int \frac{\sin x}{\cos x}\,dx$ $\displaystyle\int \frac{\sec^2 x}{\tan x}\,dx$ $\displaystyle\int \frac{\operatorname{cosec}^2 x}{\cot x}\,dx$

$\displaystyle\int \frac{e^x}{e^x+1}\,dx$ $\displaystyle\int \frac{3^x}{3^x+1}\,dx$ $\displaystyle\int \frac{5^x}{5^x-1}\,dx$ $\displaystyle\int \frac{2^x}{2^x+3}\,dx$

สูตร 3 $\int \sin u\,du = -\cos u + C$

จงหาอินทิกรัลต่อไปนี้

$\int \sin 2x\,dx$ $\int \sin(2x+1)\,dx$ $\int x\sin(x^2)\,dx$ $\int x^2\sin(x^3)\,dx$

$\int \frac{\sin(\ln x)}{x}\,dx$ $\int e^x\sin(e^x)\,dx$ $\int e^x\sin(e^x+1)\,dx$ $\int 3^x\sin(3^x)\,dx$

สูตร 4 $\int \cos u\,du = \sin u + C$

จงหาอินทิกรัลต่อไปนี้

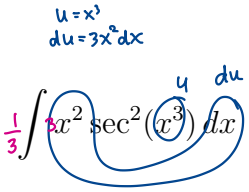
$\int \cos 2x\,dx$ $\int \cos(2x+1)\,dx$ $\int x\cos(x^2)\,dx$ $\int x^2\cos(x^3)\,dx$

$\int \frac{\cos(\ln x)}{x}\,dx$ $\int e^x\cos(e^x)\,dx$ $\int e^x\cos(e^x+1)\,dx$ $\int 3^x\cos(3^x)\,dx$

สูตร 5 $\int \sec^2 u\,du = \tan u + C$

จงหาอินทิกรัลต่อไปนี้

$\int \sec^2 2x\,dx$ $\int \sec^2(2x+1)\,dx$ $\int x\sec^2(x^2)\,dx$ $\int 3x^2\sec^2(x^3)\,dx$



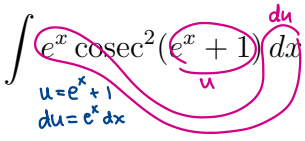
$\int \frac{\sec^2(\ln x)}{x}\,dx$ $\int e^x\sec^2(e^x)\,dx$ $\int e^x\sec^2(e^x+1)\,dx$ $\int 3^x\sec^2(3^x)\,dx$

สูตร 6 $\int \operatorname{cosec}^2 u\,du = -\cot u + C$

จงหาอินทิกรัลต่อไปนี้

$\int \operatorname{cosec}^2 2x\,dx$ $\int \operatorname{cosec}^2(2x+1)\,dx$ $\int x\operatorname{cosec}^2(x^2)\,dx$ $\int x^2\operatorname{cosec}^2(x^3)\,dx$

$\int \frac{\operatorname{cosec}^2(\ln x)}{x}\,dx$ $\int e^x\operatorname{cosec}^2(e^x)\,dx$



$\int 3^x\operatorname{cosec}^2(3^x)\,dx$

สูตร 7 $\int \sec u \tan u \, du = \sec u + C$

จงหาอินทิกรัลต่อไปนี้

$\int \sec 2x \tan 2x \, dx$ $\int \frac{\sec(\ln x)\tan(\ln x)}{x} \, dx$ $\int e^x \sec e^x \tan e^x \, dx$

สูตร 8 $\int \operatorname{cosec} u \cot u \, du = -\operatorname{cosec} u + C$

จงหาอินทิกรัลต่อไปนี้

$u = \ln x$, $du = \frac{1}{x}dx$ } $\int \operatorname{cosec} u \cot u \, du$

$\int \operatorname{cosec} 2x \cot 2x \, dx$ $\int \frac{\operatorname{cosec}(\ln x)\cot(\ln x)}{x} \, dx$ $\int e^x \operatorname{cosec} e^x \cot e^x \, dx$

สูตร 9 $\int \tan u \, du = \ln|\sec u| + C$

จงหาอินทิกรัลต่อไปนี้

$\int \tan 2x \, dx$ $\int \tan(2x+1) \, dx$ $\int x \tan(x^2) \, dx$ $\int x^2 \tan(x^3) \, dx$

$\int \frac{\tan(\ln x)}{x} \, dx$ $\int e^x \tan(e^x) \, dx$ $\int e^x \tan(e^x+1) \, dx$ $\int 7^x \tan(7^x) \, dx$

สูตร 10 $\int \cot u \, du = \ln|\sin u| + C$

จงหาอินทิกรัลต่อไปนี้

$\int \cot 2x \, dx$ $\int \cot(2x+1) \, dx$ $\int x \cot(x^2) \, dx$ $\int x^2 \cot(x^3) \, dx$

$\int \frac{\cot(\ln x)}{x} \, dx$ $\int e^x \cot(e^x) \, dx$ $\int e^x \cot(e^x+1) \, dx$ $\int 2^x \cot(2^x) \, dx$

สูตร 11 $\int \sec u \, du = \ln|\sec u + \tan u| + C$

จงหาอินทิกรัลต่อไปนี้

$\int \sec 2x \, dx$ $\int \sec(2x+1) \, dx$ $\int x \sec(x^2) \, dx$ $\int x^2 \sec(x^3) \, dx$

$\int \frac{\sec(\ln x)}{x} \, dx$ $\int e^x \sec(e^x) \, dx$ $\int e^x \sec(e^x+1) \, dx$ $\int 3^x \sec(3^x) \, dx$

สูตร 12 $\displaystyle\int \operatorname{cosec} u\,du = \ln|\operatorname{cosec} u - \cot u| + C$

จงหาอินทิกรัลต่อไปนี้

$\displaystyle\int \operatorname{cosec} 2x\,dx$ $\displaystyle\int \operatorname{cosec}(2x+1)\,dx$ $\displaystyle\int x\operatorname{cosec}(x^2)\,dx$ $\displaystyle\int x^2\operatorname{cosec}(x^3)\,dx$

$\displaystyle\int \frac{\operatorname{cosec}(\ln x)}{x}\,dx$ $\displaystyle\int e^x\operatorname{cosec}(e^x)\,dx$ $\displaystyle\int e^x\operatorname{cosec}(e^x+1)\,dx$ $\displaystyle\int 3^x\operatorname{cosec}(3^x)\,dx$

สูตร 13 $\displaystyle\int e^u\,du = e^u + C$

จงหาอินทิกรัลต่อไปนี้

$\displaystyle\int e^{2x}\,dx$ $\displaystyle\int e^{-x}\,dx$ $\displaystyle\int e^{-3x}\,dx$ $\displaystyle\int e^{2x+1}\,dx$ $\displaystyle\int xe^{x^2}\,dx$ $\displaystyle\int x^2e^{x^3}\,dx$

$\displaystyle\int e^{\sin x}\cos x\,dx$ $\displaystyle\int e^{\cos x}\sin x\,dx$ $\displaystyle\int e^{\tan x}\sec^2 x\,dx$ $\displaystyle\int e^{\cot x}\operatorname{cosec}^2 x\,dx$

$\displaystyle\int e^{\sec x}\sec x\tan x\,dx$ $\displaystyle-\int e^{\operatorname{cosec} x}\operatorname{cosec} x\cot x\,dx$ $\displaystyle\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx$ $\displaystyle\int \frac{e^{\arctan x}}{1+x^2}\,dx$

u = cosec x
du = −cosec x cot x dx

สูตร 14 $\displaystyle\int a^u\,du = \frac{a^u}{\ln a} + C$

จงหาอินทิกรัลต่อไปนี้

$\displaystyle\int 2^{3x}\,dx$ $\displaystyle\int 2^{-x}\,dx$ $\displaystyle\int 2^{-3x}\,dx$ $\displaystyle\int 2^{2x+1}\,dx$ $\displaystyle\int x2^{x^2}\,dx$ $\displaystyle\int x^22^{x^3}\,dx$

$\displaystyle\int 2^{\sin x}\cos x\,dx$ $\displaystyle\int 3^{\cos x}\sin x\,dx$ $\displaystyle\int 7^{\tan x}\sec^2 x\,dx$ $\displaystyle\int 5^{\cot x}\operatorname{cosec}^2 x\,dx$

$\displaystyle\int 7^{\sec x}\sec x\tan x\,dx$ $\displaystyle\int 2^{\operatorname{cosec} x}\operatorname{cosec} x\cot x\,dx$ $\displaystyle\int \frac{3^{\arcsin x}}{\sqrt{1-x^2}}\,dx$ $\displaystyle\int \frac{5^{\arctan x}}{1+x^2}\,dx$

สูตร 15 $\displaystyle\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\frac{u}{a}+C$

จงหาอินทิกรัลต่อไปนี้

$\displaystyle\int \frac{1}{\sqrt{5-x^2}}\,dx$ $\displaystyle\int \frac{x}{\sqrt{4-x^4}}\,dx$ $\displaystyle\int \frac{x^2}{\sqrt{9-x^6}}\,dx$ $\displaystyle\int \frac{1}{x\sqrt{5-(\ln x)^2}}\,dx$

$\displaystyle\int \frac{\cos x}{\sqrt{2-\sin^2 x}}\,dx$ $\displaystyle\int \frac{\sin x}{\sqrt{3-\cos^2 x}}\,dx$ $\displaystyle\int \frac{\sec^2 x}{\sqrt{9-\tan^2 x}}\,dx$ $\displaystyle\int \frac{\operatorname{cosec}^2 x}{\sqrt{4-\cot^2 x}}\,dx$

$\displaystyle\int \frac{e^x}{\sqrt{2-e^{2x}}}\,dx$ $\displaystyle\int \frac{3^x}{\sqrt{5-3^{2x}}}\,dx$ $\displaystyle\int \frac{\sec x\tan x}{\sqrt{4-\sec^2 x}}\,dx$ $\displaystyle\int \frac{\operatorname{cosec} x\cot x}{\sqrt{9-\operatorname{cosec}^2 x}}\,dx$

สูตร 16 $\displaystyle\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\frac{u}{a}+C$

จงหาอินทิกรัลต่อไปนี้

$\displaystyle\int \frac{1}{5+x^2}\,dx$ $\displaystyle\int \frac{x}{4+x^4}\,dx$ $\displaystyle\int \frac{x^2}{9+x^6}\,dx$ $\displaystyle\int \frac{1}{x(5+(\ln x)^2)}\,dx$

$\displaystyle\int \frac{\cos x}{2+\sin^2 x}\,dx$ $\displaystyle\int \frac{\sin x}{3+\cos^2 x}\,dx$ $\displaystyle\int \frac{\sec^2 x}{9+\tan^2 x}\,dx$ $\displaystyle\int \frac{\operatorname{cosec}^2 x}{4-\cot^2 x}\,dx$

$\displaystyle\int \frac{e^x}{2+e^{2x}}\,dx$ $\displaystyle\int \frac{3^x}{5+3^{2x}}\,dx$ $\displaystyle\int \frac{\sec x\tan x}{4+\sec^2 x}\,dx$ $\displaystyle\int \frac{\operatorname{cosec} x\cot x}{9+\operatorname{cosec}^2 x}\,dx$