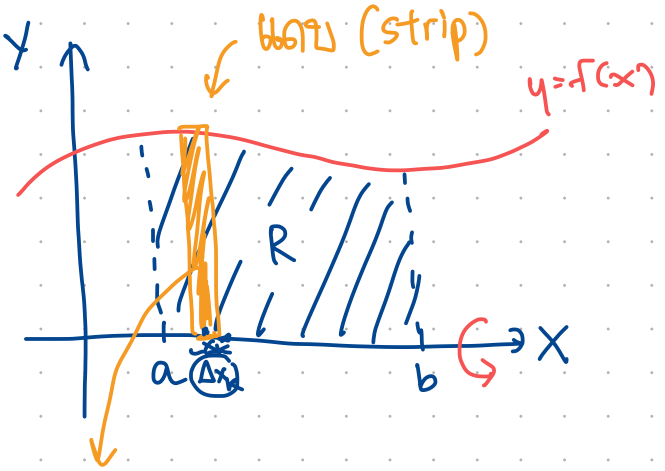
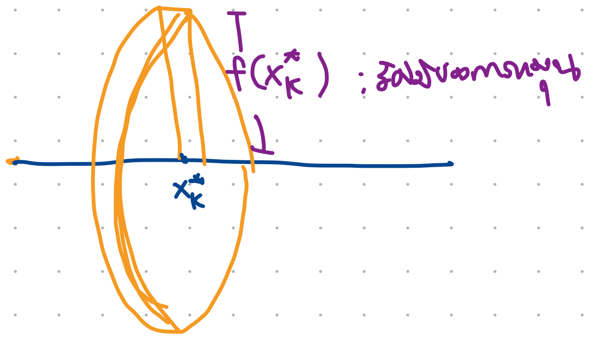
จานกลม



$$V = \int_a^b A(x)\,dx$$



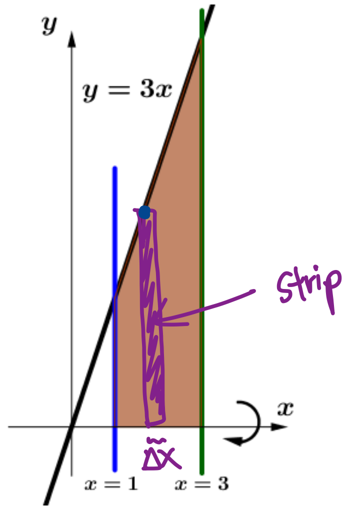
$$\Rightarrow A(x_k^*) = \pi\,[f(x_k^*)]^2$$

$$\therefore V = \int_a^b \pi\,[f(x)]^2\,dx$$

Disk method

Because the cross sections are disk shaped, the application of this formula is called the ***method of disks.***

Example 8.6 Find the volume of the solid that is obtained when the region under the curve $y = 3x$ over the interval [1, 3] is revolved about the X-axis.



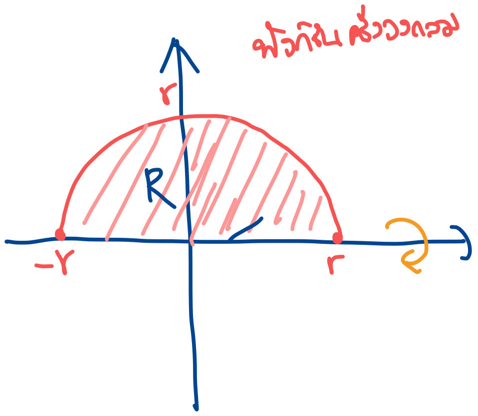
$$V = \int_1^3 \pi [3x]^2 \, dx$$

$$V = \pi \int_1^3 9x^2 \, dx = \pi \left[9 \cdot \frac{x^3}{3} \right]_1^3 = 3\pi [27 - 1] = (3\pi)(26) = 78\pi$$

Challenge problem

$$V = \frac{4}{3}\pi r^3$$

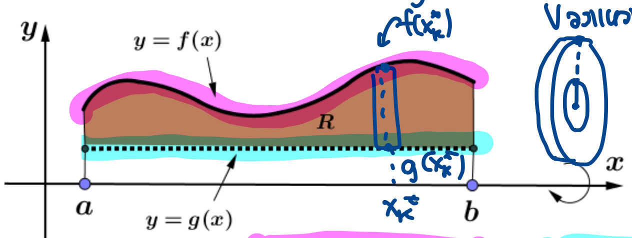
$f(x) = \sqrt{r^2 - x^2}$

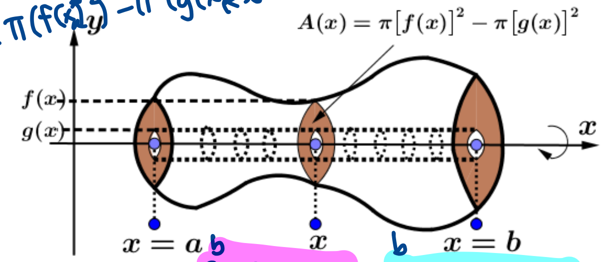


$$V = \frac{4}{3}\pi r^3$$

Volume by Washers Perpendicular to the X-axis

Problem: Let f and g be continuous and nonnegative on $[a, b]$, and suppose that $f(x) \geq g(x)$ for all x in the interval $[a, b]$. Let R be the region that is bounded above by $y = f(x)$, below by $y = g(x)$, and on the sides by the lines $x = a$ and $x = b$. Find the volume of the solid of revolution that is generated by revolving the region R about the X-axis.





We can solve this problem by slicing. For this purpose, observe that the cross section of the solid taken perpendicular to the X-axis at the point x is the annular or "washer-shaped", region with inner radius $g(x)$ and outer radius $f(x)$. The area of this region is

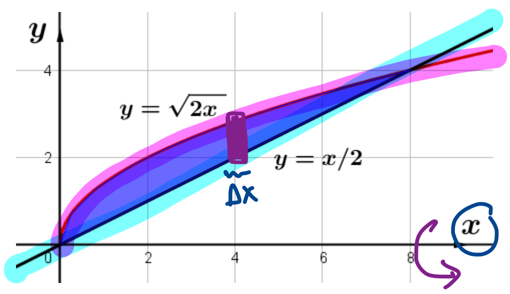
$$A(x) = \pi[f(x)]^2 - \pi[g(x)]^2 = \pi\left([f(x)]^2 - [g(x)]^2\right)$$

Thus, from (8.3) the volume of the solid is

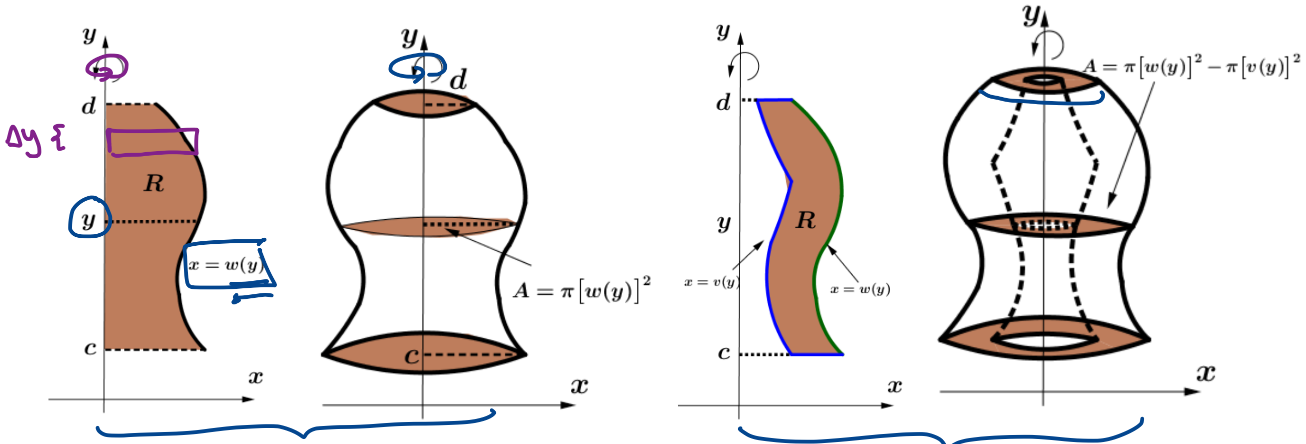
$$V = \int_a^b \pi\left([f(x)]^2 - [g(x)]^2\right) dx \tag{8.6}$$

Because the cross sections are washer shaped, the application of this formula is called the *method of washers.*

Example 8.7 Find the volume of the solid that is obtained when the region between the graphs of the equations $y = \sqrt{2x}$ and $y = \frac{x}{2}$ over the interval $[0, 8]$ is revolved about the X-axis.



Volume by Disks and Washers perpendicular to the Y-axis

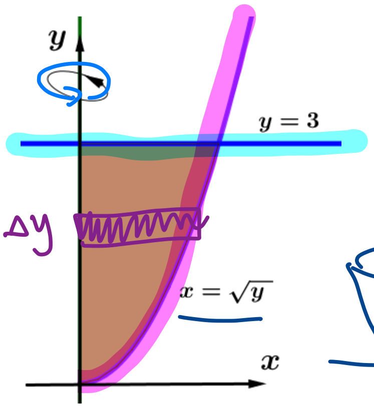


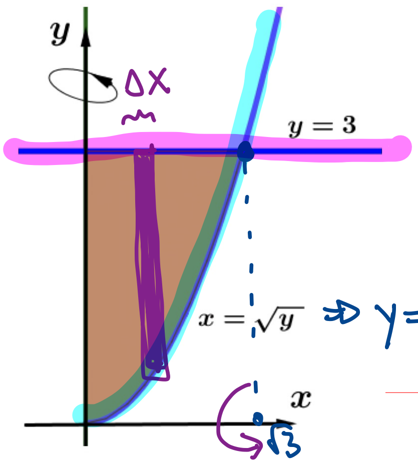
The methods of disks and washers have analogs for regions that are revolved about the Y-axis. Using the method of slicing and Formula (8.4), the following formulas for the volumes of the solid are

$$V = \int_c^d \pi[w(y)]^2 dy \qquad (disks), \tag{8.7}$$

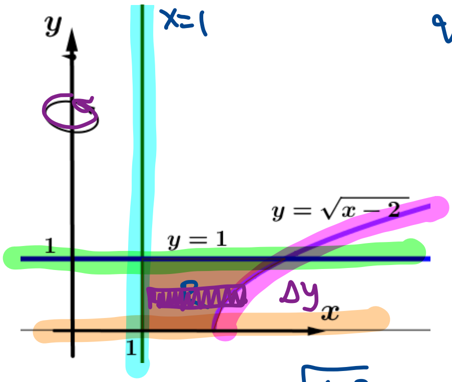
$$V = \int_c^d \pi\left([w(y)]^2 - [v(y)]^2\right) dy \qquad (washers). \tag{8.8}$$

Example 8.8 Find the volume of the solid generated when the region enclosed by $x = \sqrt{y}$, $x = 0$, and $y = 3$ is revolved about the Y-axis.





Example 8.9 Find the volume of the solid generated when the region enclosed by $x = 1$, $y = \sqrt{x-2}$, $y = 0$, and $y = 1$ is revolved about the Y-axis.



หมุนแบบ washer รอบแกน y.

$$V = \int_0^1 \pi[(y^2+2)^2 - (1)^2]\,dy$$

$$= \pi\int_0^1 [y^4+4y^2+4-1]\,dy$$

$$= \pi\int_0^1 [y^4+4y^2+3]\,dy$$

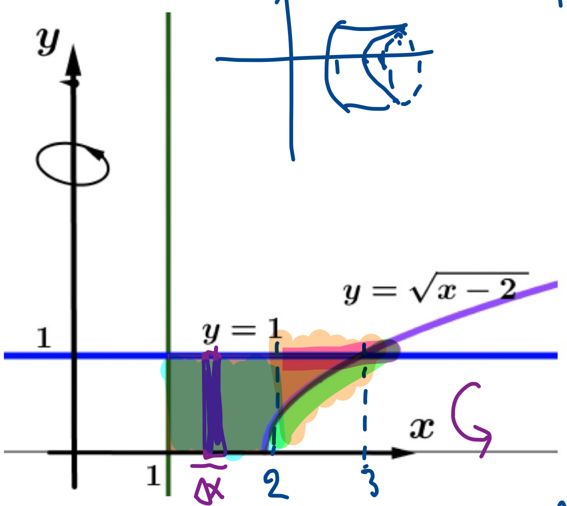
$$= \pi\left[\frac{y^5}{5} + \frac{4y^3}{3} + 3y\right]_0^1$$

$$= \pi\left[\frac{1}{5}+\frac{4}{3}+3\right] = \pi\left(\frac{3+20+45}{15}\right) = \frac{68}{15}\pi$$

$y = \sqrt{x-2}$, $y^2 = x-2$, $\therefore x = y^2+2$

หมุนรอบแกน x ด้วย disk / washer

$$V = \int_1^2 \pi[1]^2\,dx + \int_2^3 \pi[(1)^2 - (\sqrt{x-2})^2]\,dx$$



$y = \sqrt{x-2}$, $y = 1$, $1 = \sqrt{x-2}$, $x = 3$

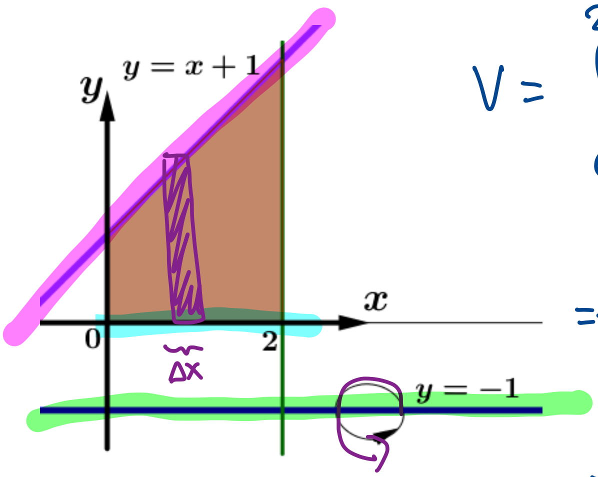
$V = \frac{3\pi}{2}$

Other axes of revolution

It is possible to use the method of disks and the method of washers to find the volume of a solid of revolution whose axis of revolution is a line other than one of the coordinate axes. Instead of developing a new formula for each situation, we will appeal to Formulas (8.3) and (8.4) and integrate an appropriate cross-sectional area to find the volume.

washer รอบ $y=-1$

Example 8.10 Find the volume of the solid that is obtained when the region between the curve $y = x + 1$ and $y = 0$ over the interval $[0, 2]$ is rotated about the line $y = -1$.



$$V = \int_0^2 \pi\left[\{(x+1)-(-1)\}^2 - \{0-(-1)\}^2\right]dx$$

(function, ฟังก์ชัน, แกนหมุน, แกนหมุน)

$$= \pi\int_0^2 \left[(x+2)^2 - (1)^2\right]dx$$

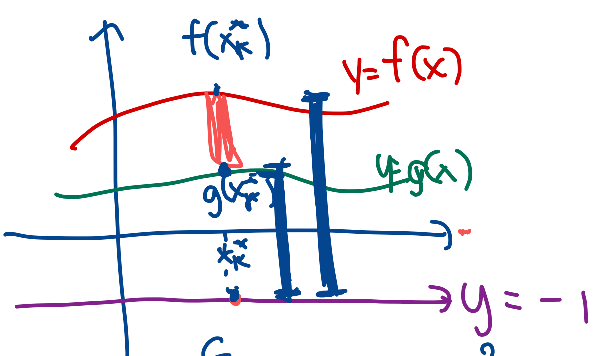
$$= \pi\int_0^2 \left[x^2+4x+4-1\right]dx$$

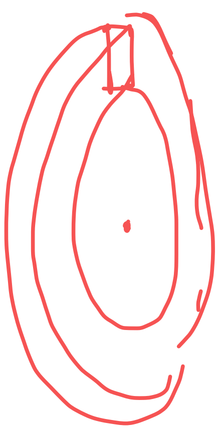
$$= \pi\int_0^2 \left[x^2+4x+3\right]dx$$

$$= \pi\left[\frac{x^3}{3}+\frac{4x^2}{2}+3x\right]_0^2$$

$$= \pi\left[\frac{8}{3}+8+6\right] = \pi\left[\frac{8+24+18}{3}\right]$$

$$= \frac{50}{3}\pi$$

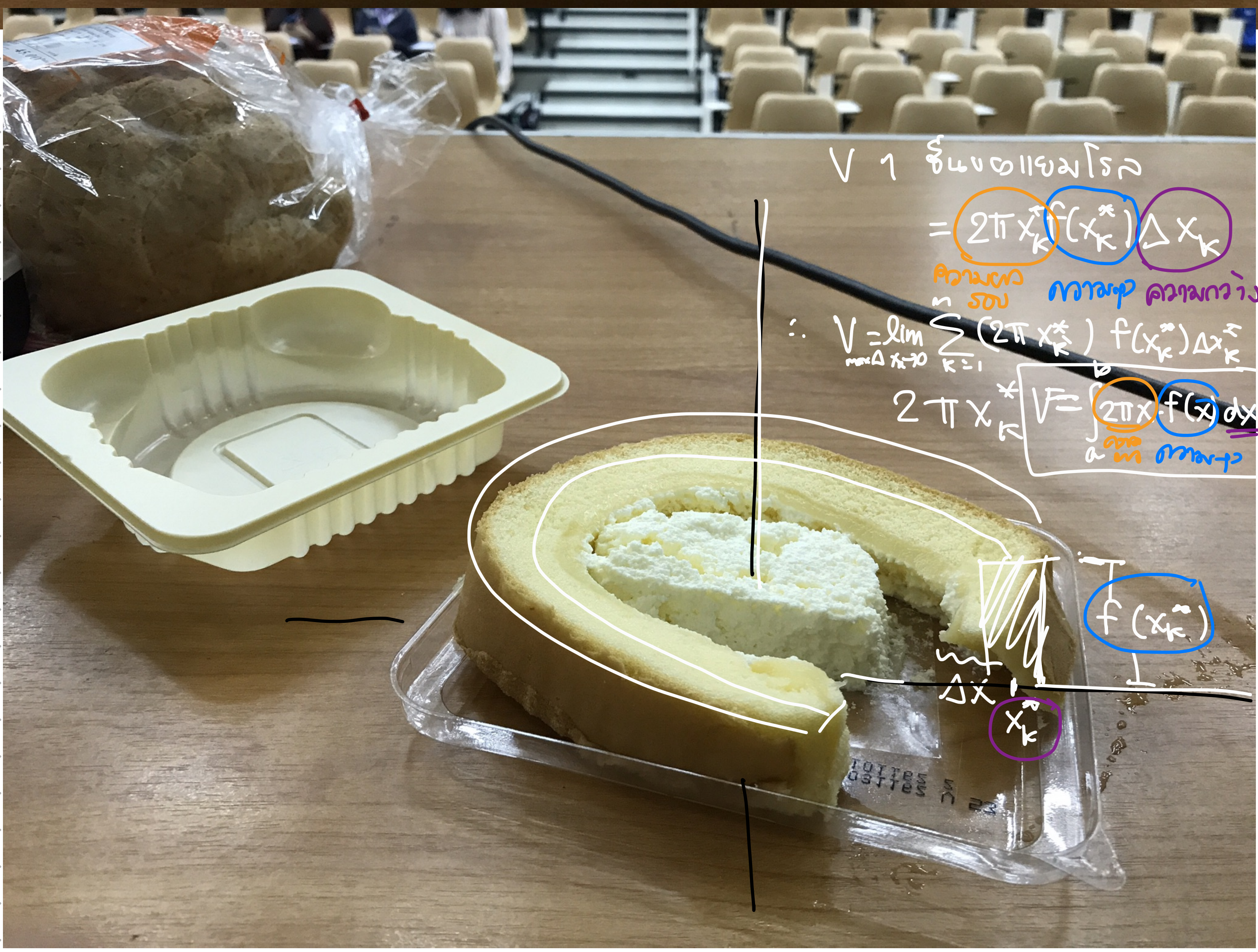




$$V_k = \pi\left\{\left[f(x_k^*)-(-1)\right]^2 - \left[g(x_k^*)-(-1)\right]^2\right\}$$

รัศมี

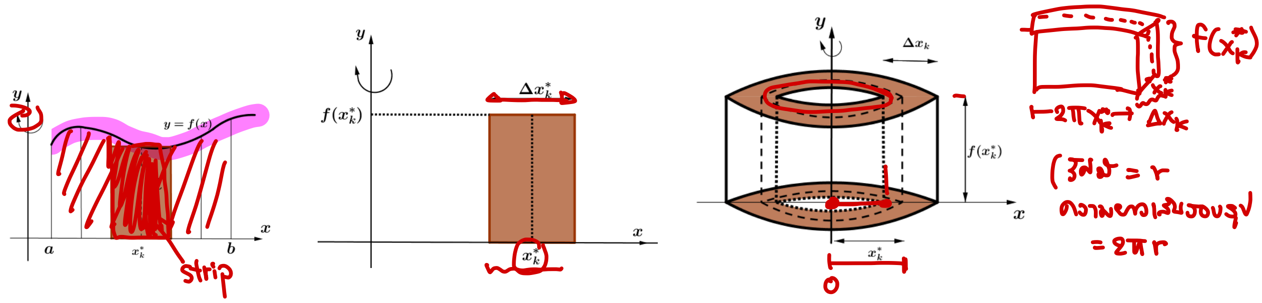

หาปริมาตรแยมโรล


V 1 ชิ้นของแยมโรล
$= 2\pi x_k^* f(x_k^*) \Delta x_k$
ความยาวรอบ
ความสูง
ความกว้าง
$\therefore V = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} (2\pi x_k^*) f(x_k^*) \Delta x_k^*$
$2\pi x_k^*$
$V = \int_a^b 2\pi x f(x) dx$
ความยาวรอบ
ความสูง
$f(x_k^*)$
Δx
x_k^*

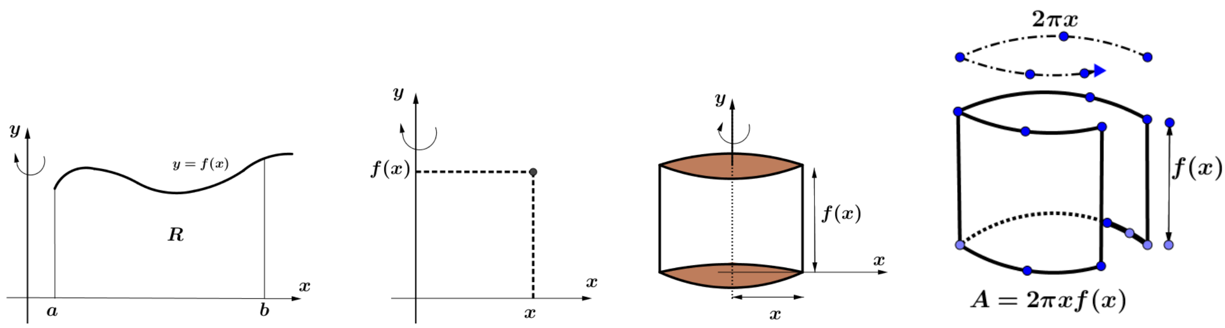
8.3 Volumes by Cylindrical Shells

Theorem 8.5 (**Volume by cylindrical shells about the Y-axis**) Let f be continuous and nonnegative on [a, b] and let R be the region that is bounded above by $y = f(x)$, below by the X-axis, and on the sides by the lines $x = a$ and $x = b$. Then the volume V of the solid of revolution that is generated by revolving the region R about the Y-axis is given by

$$V = \int_a^b 2\pi x f(x)dx. \tag{8.9}$$



$$V = \lim_{max \Delta x_k \to 0} \sum_{k=1}^{n} 2\pi x_k^* f(x_k^*)\Delta x_k = \int_a^b 2\pi x f(x)dx.$$



Example 8.11 Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^2$, $x = 1$, $x = 2$ and the X-axis is revolved about the Y-axis.