$$Ax^2+By^2+Cz^2+\underline{Dxy}+\underline{Eyz}+\underline{Fxz}+Gx+Hy+Iz+J=0$$

$$\boxed{Ax^2+By^2+Cz^2+Gx+Hy+Iz+J=0}$$

① Ellipsoid : $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

② hyperboloid one sheet $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

③ hyperboloid two sheets : $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

---

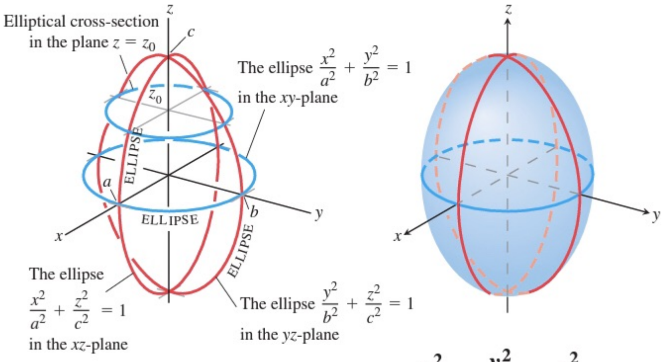
④ elliptic cone $z^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

⑤ elliptic paraboloid $z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

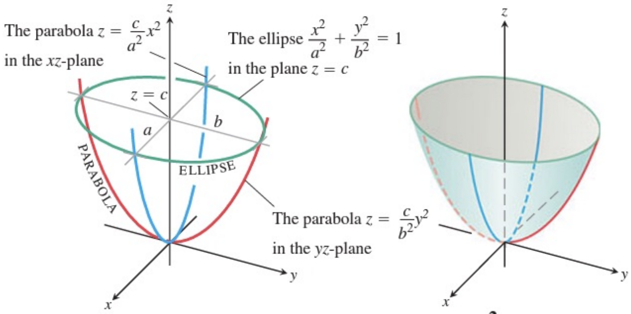
⑥ hyperbolic paraboloid $z=\frac{y^2}{b^2}-\frac{x^2}{a^2}$

206112 QUICK REFERENCE

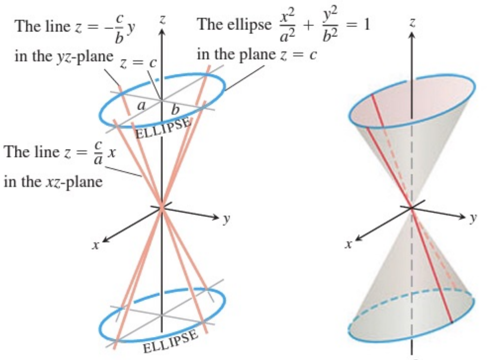
Quadric Surfaces



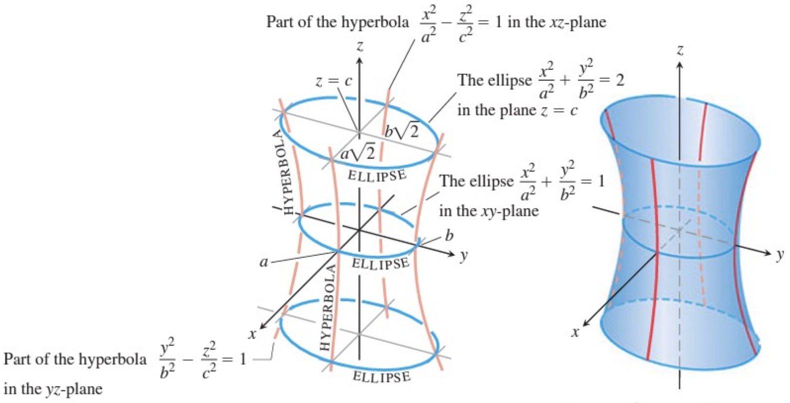
ELLIPSOID $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$



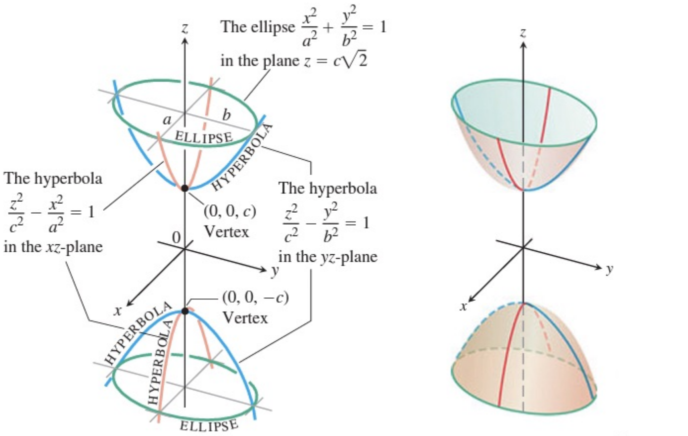
ELLIPTICAL PARABOLOID $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{z}{c}$



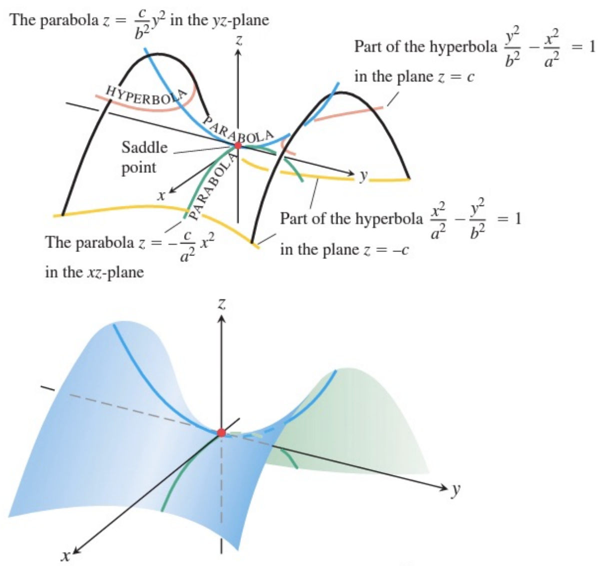
ELLIPTICAL CONE $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{z^2}{c^2}$



HYPERBOLOID OF ONE SHEET $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$



HYPERBOLOID OF TWO SHEETS $\dfrac{z^2}{c^2} - \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$



HYPERBOLIC PARABOLOID $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = \dfrac{z}{c}, c > 0$

ที่มา : หนังสือ Thomas Calculus

A rough sketch of the **hyperboloid of two sheet**

$$\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad (a>0, b>0, c>0)$$

can be obtained by first plotting the intersections with the z-axis, then sketching the elliptical traces in the planes $z=\pm 2c$, and then sketching the hyperbolic traces that connect the z-axis intersections and the endpoints of the axes of the ellipses. (It is not essential to use the planes $z=\pm 2c$, but these are good choices since they simplify the calculations slightly and have the right spacing for a good sketch.) The next example illustrates this technique.

Example 9 Sketch the graph of the hyperboloid of two sheet $z^2-x^2-\frac{y^2}{4}=1$

A rough sketch of the **elliptic cone** $z^2 - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

$$z^2=\frac{x^2}{a^2}+\frac{y^2}{b^2} \qquad (a>0, b>0)$$

can be obtained by first sketching the elliptical traces in the planes $z=\pm 1$ and then sketching the linear traces that connect the endpoints of the axes of the ellipses. The next example illustrates this technique.

Example 10 Sketch the graph of the elliptic cone $z^2=x^2+\frac{y^2}{4}$

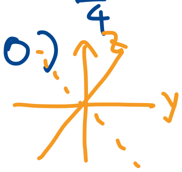
ถ้า $z=0$ ได้ $0 = x^2+\frac{y^2}{4}$ เป็นจุด

$z=1$ $1 = x^2+\frac{y^2}{4}$ เป็นวงรี

$z=-1$ $(-1)^2 = x^2+\frac{y^2}{4}$ เป็นวงรี

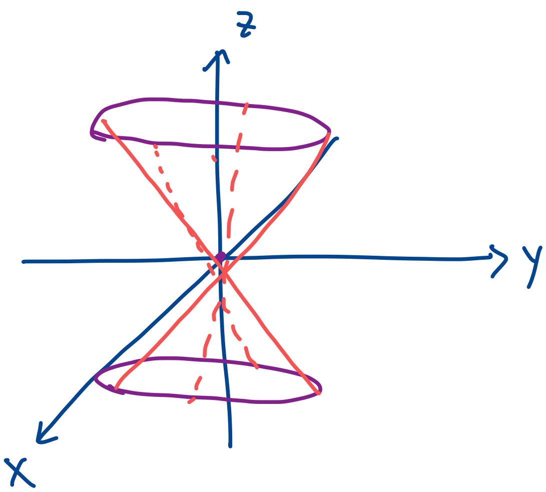
CHECK ตัดระนาบ YZ (set $x=0$)

จะได้ $z^2 = \frac{y^2}{4}$



$z^2 - \frac{y^2}{4} = 0$

$(z-\frac{y}{2})(z+\frac{y}{2})=0 \iff z = \frac{y}{2}, \ z = -\frac{y}{2}$



A rough sketch of the **elliptic paraboloid**

$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2} \qquad (a > 0, b > 0)$$

can be obtained by first sketching the elliptical trace in the plane $z = 1$ and then sketching the parabolic traces in the vertical coordinate planes to connect the origin to the ends of the axes of the ellipse. The next example illustrates this technique.

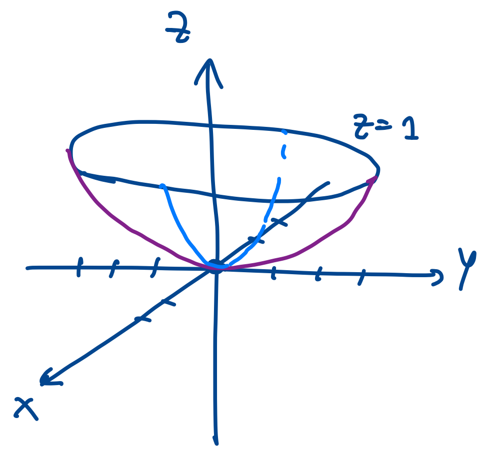
Example 11 Sketch the graph of the elliptic paraboloid $z = \frac{x^2}{4} + \frac{y^2}{9}$

$z = 0$ เกิดจุด

$z = 1$ เกิดวงรี $\quad 1 = \frac{x^2}{4} + \frac{y^2}{9}$

ตัดตาม yz (set $x=0$: $z = \frac{y^2}{9}$ พาราโบลา

ตัดตาม xz (set $y=0$: $z = \frac{x^2}{4}$ พาราโบลา)



A rough sketch of the **hyperbolic paraboloid**

$$z^2 = \frac{y^2}{b^2} - \frac{x^2}{a^2} \qquad (a > 0, b > 0)$$

can be obtained by first sketching the two parabolic traces that pass through the origin (one in the plane $x = 0$ and the other in the plane $y = 0$). After the parabolic traces are drawn, sketch the hyperbolic traces in the planes $z = \pm 1$ and then fill in any missing edges. The next example illustrates this technique.

Example 12 Sketch the graph of the hyperbolic paraboloid $z = \frac{y^2}{4} - \frac{x^2}{9}$

① หารอยตัดตามระนาบ xz, yz ($y=0$) ($x=0$)

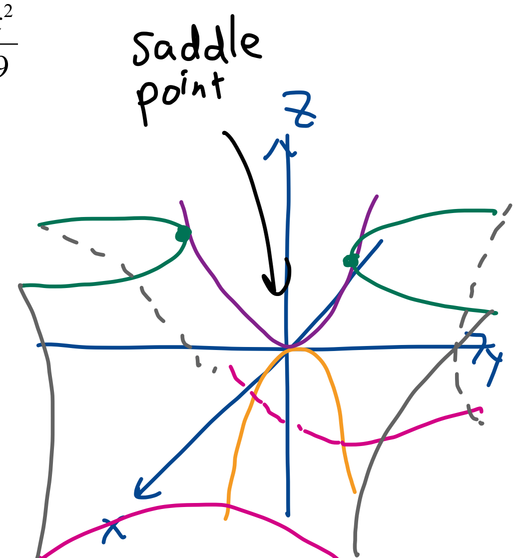
$y=0$ ได้ $z = -\frac{x^2}{9}$

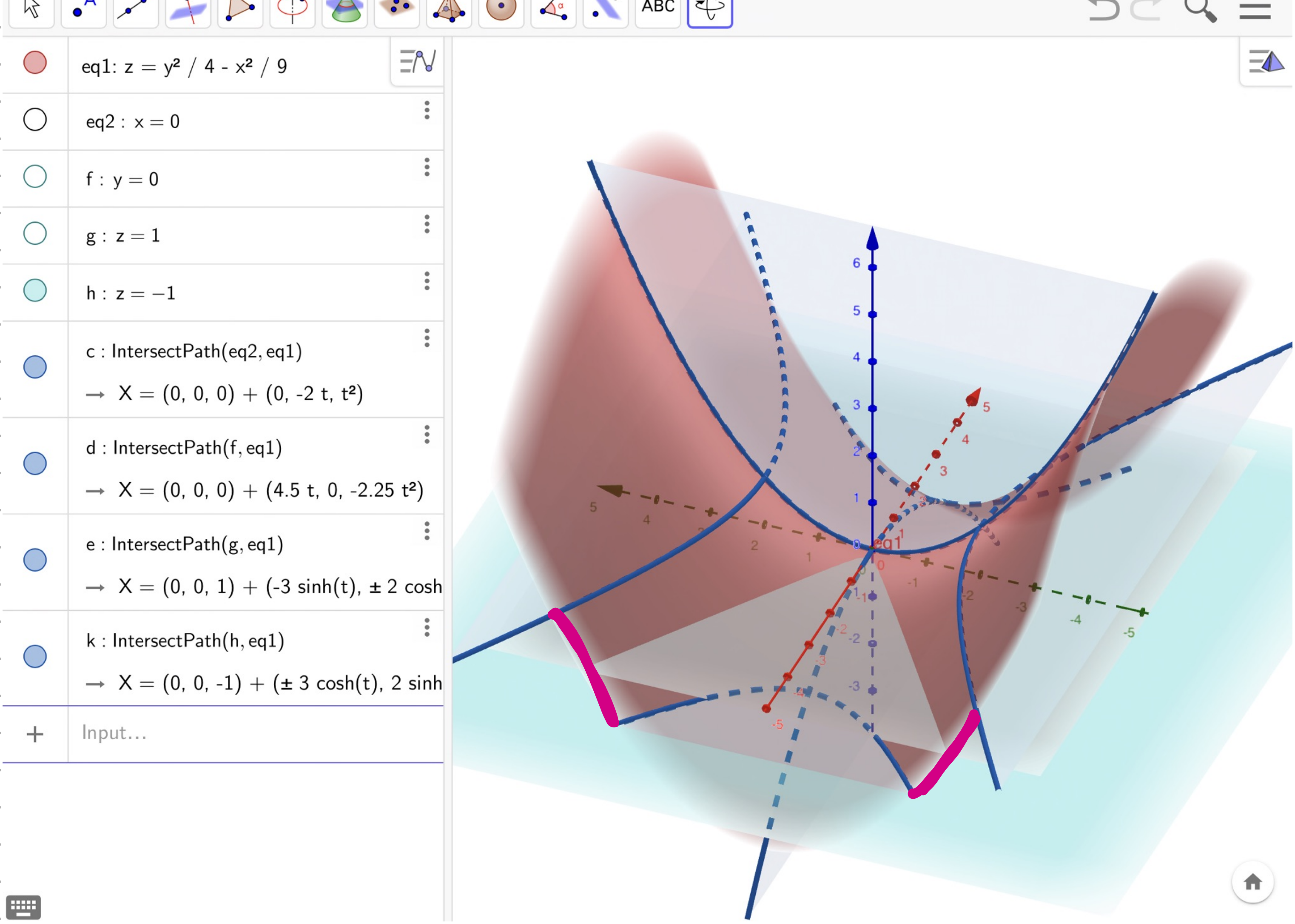
$x=0$ ได้ $z = \frac{y^2}{4}$

② หารอยตัด hyperbola ตามแกน z ($z = \pm 1$)

$z=1$ ได้ $1 = \frac{y^2}{4} - \frac{x^2}{9}$

$z=-1$ ได้ $-1 = \frac{y^2}{4} - \frac{x^2}{9} \Leftrightarrow \frac{x^2}{9} - \frac{y^2}{4} = 1$

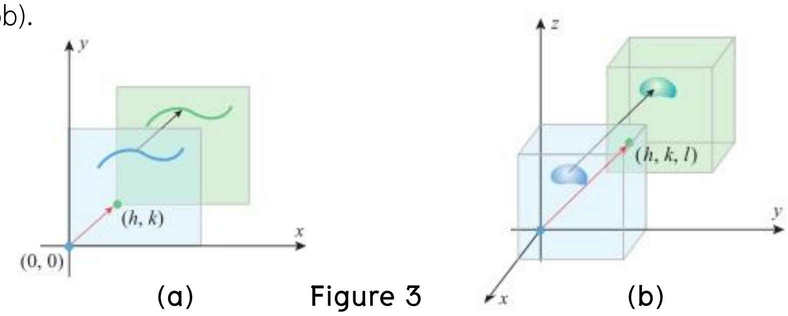



ABC
eq1: z = y² / 4 - x² / 9
eq2 : x = 0
f : y = 0
g : z = 1
h : z = −1
c : IntersectPath(eq2, eq1)
→ X = (0, 0, 0) + (0, -2 t, t²)
d : IntersectPath(f, eq1)
→ X = (0, 0, 0) + (4.5 t, 0, -2.25 t²)
e : IntersectPath(g, eq1)
→ X = (0, 0, 1) + (-3 sinh(t), ± 2 cosh
k : IntersectPath(h, eq1)
→ X = (0, 0, -1) + (± 3 cosh(t), 2 sinh
Input...
eq1

5.2 Translations of quadric surfaces

A conic in an xy–coordinate system can be translated by substituting $x-h$ for x and $y-k$ for y in its equation. To understand why this works, think of the xy–axes as fixed and think of the plane as a transparent sheet of plastic on which all graphs are drawn. When the coordinates of points are modified by substituting $(x-h, y-k)$ for (x, y), the geometric effect is to translate the sheet of plastic (and hence all curves) so that the point on the plastic that was initially at $(0,0)$ is moved to the point (h,k) (see Figure 3a).

For the analog in three dimensions, think of the xyz–axes as fixed and think of 3–space as a transparent block of plastic in which all surfaces are embedded. When the coordinates of points are modified by substituting $(x-h, y-k, z-l)$ for (x, y, z) the geometric effect is to translate the block of plastic (and hence all surfaces) so that the point in the plastic block that was initially at $(0,0,0)$ is moved to the point (h,k,l) (see Figure 3b).



(a) Figure 3 (b)

(From: Calculus Early Transcendentals, 10th edition, Howard Anton, Irl C. Bevens, Stephen Davis, page 827)

$z-3$ x^2 y^2

Example 13 Describe the surface $z = (x-1)^2 + (y+2)^2 + 3$

$$z - 3 = (\boxed{x-1})^2 + (y+2)^2 \qquad \heartsuit = \square^2 + \bigcirc^2$$

เกิดจากการเลื่อน elliptic paraboloid $z = x^2 + y^2$ ไปยังจุด $(1, -2, 3)$

$\therefore$ จุดยอดคือ $(1, -2, 3)$

Example 14 Describe the surface $4x^2 + 4y^2 + z^2 + 8y - 4z = -4$ ← พยายามจัดกำลังสองสมบูรณ์

$$4x^2 + (4y^2 + 8y) + (z^2 - 4z) = -4$$

$$4x^2 + 4(y^2 + 2y + 1) + (z^2 - 4z + 4) = -4 + 4 + 4$$

$$4x^2 + 4(y+1)^2 + (z-2)^2 = 4$$

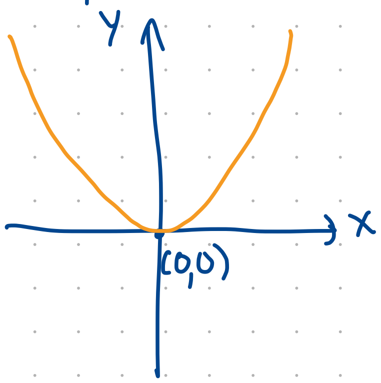
$$x^2 + (y+1)^2 + \frac{(z-2)^2}{4} = 1$$

เป็น ellipsoid ที่ย้าย $x^2 + y^2 + \frac{z^2}{4}$ ไปที่จุด $(0, -1, 2)$

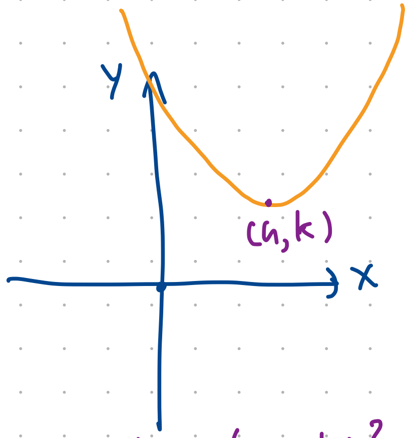
ตามแกน z

parabola

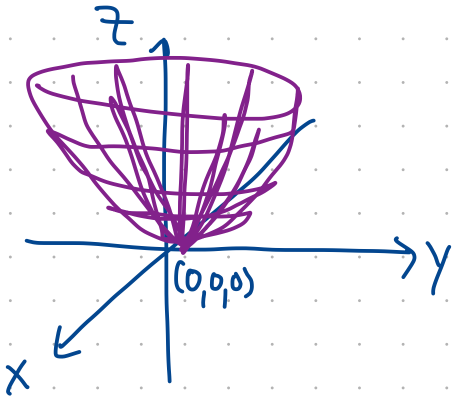
$y = x^2$



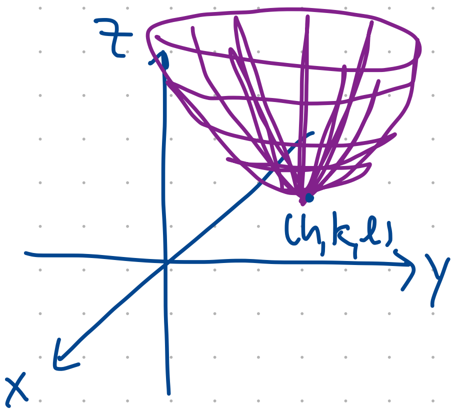
ย้ายจุดยอดไปยัง (h,k)



$y-k = (x-h)^2$



ย้ายจุดยอดไปยัง (h,k,l)



*จัดรูปสมการให้อยู่ในรูปมาตรฐาน $(x-h,\ y-k,\ z-l)$

5.2 Techniques for identifying quadric surfaces

The equations of the quadric surfaces in Table 1 have certain characteristics that make it possible to identify quadric surfaces that are derived from these equations by reflections. These identifying characteristics, which are shown in Table 2, are based on writing the equation of the quadric surface so that all of the variable terms are on the left side of the equation and there is a 1 or a 0 on the right side. These characteristics do not change when the surface is reflected about a coordinate plane or planes of the form $x = y, x = z,$ or $y = z,$ thereby making it possible to identify the reflected quadric surface from the form of its equation.

Table 2

identifying a quadric surface from the form of its equation

equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	$\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$z^2-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$	$z-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$	$z-\frac{y^2}{b^2}+\frac{x^2}{a^2}=0$
characteristic	No minus signs	One minus sign	Two minus signs	No linear terms	One linear term; two quadratic terms with the same sign	One linear term; two quadratic terms with opposite signs
classification	Ellipsoid	Hyperboloid of one sheet	Hyperboloid of two sheets	Elliptic cone	Elliptic paraboloid	Hyperbolic paraboloid

$z = \frac{x^2}{a^2}+\frac{y^2}{b^2}$

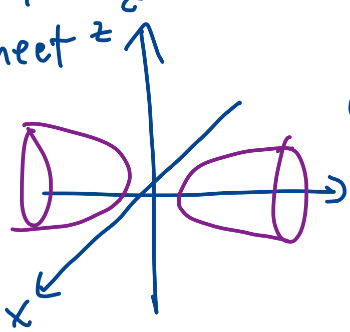
Example 15 Identify the surfaces

(a) $3x^2 - 4y^2 + 12z^2 + 12 = 0$ (b) $4x^2 - 4y + z^2 = 0$

(a) $\frac{x^2}{4} - \frac{y^2}{3} + z^2 + 1 = 0$

$\frac{y^2}{3} - \frac{x^2}{4} - \frac{z^2}{z} = 1$

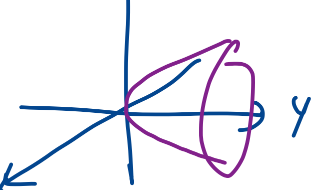
hyperboloid 2 sheet



(b) $4y = 4x^2 + z^2$

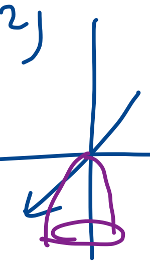
$y = x^2 + \frac{z^2}{4}$

elliptic paraboloid
ตามแกน y



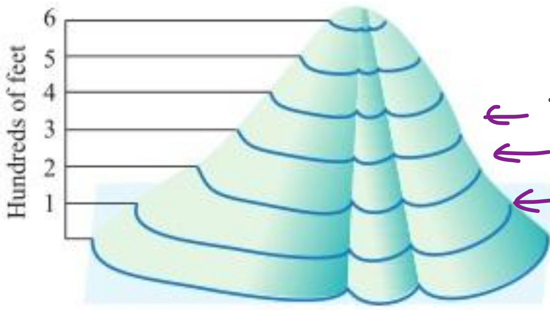
(c) $z = -x^2 - y^2;\ z = -(x^2+y^2)$

elliptic paraboloid



level curve / contour map

* ตัดตามแนวแกน $\underline{z}$ $\Leftrightarrow$ $f(x,y) = z = k$



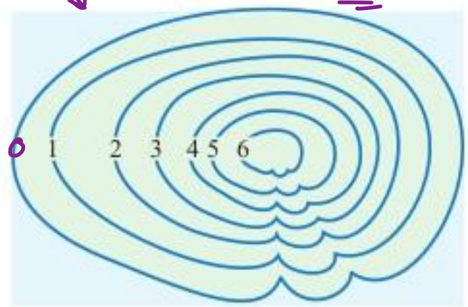
$\leftarrow z=3$

$\leftarrow z=2$

$\leftarrow z=1$

$z=0$

A perspective view of a model hill with two gullies

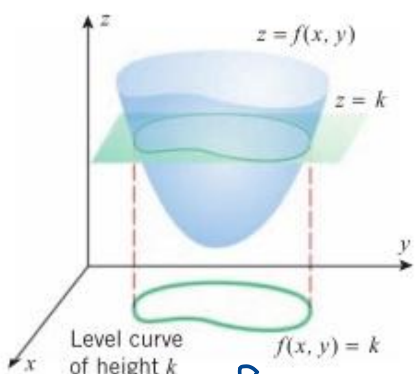


A contour map of the model hill

Figure 1

(From: Calculus Early Transcendentals, 10th edition, Howard Anton, Irl C. Bevens, Stephen Davis, page 909)

Contour maps are also useful for studying functions of two variables. If the surface $z = f(x,y)$ is cut by the horizontal plane $z = k$, then at all points on the intersection we have $f(x,y) = k$. The projection of this intersection onto the xy-plane is called the **level curve of height** k or the **level curve with constant** k (Figure 2). A set of level curves for $z = f(x,y)$ is called a **contour plot** or **contour map** of f.



Level curve of height k

ตาม k ต่างๆ

$k = 4x^2 + y^2$

$\frac{x^2}{\frac{4}{k}} + \frac{y^2}{k} = 1$ $\rightarrow$ วงรีตามแนวแกน y
จุดศก. (0,0) ความยาวแกนเอก $2\sqrt{k}$
ความยาวแกนโท $\sqrt{k}$

Figure 2

(From: Calculus Early Transcendentals, 10th edition, Howard Anton, Irl C. Bevens, Stephen Davis, page 909)

Example 5 Sketch the contour plot of $f(x,y) = 4x^2 + y^2$ using level curves of height $k = 0, 1, 2, 3, 4, 5$.

$z = 4x^2 + y^2$

Vary $\underline{z = k}$: $k=0$; $0 = 4x^2 + y^2$

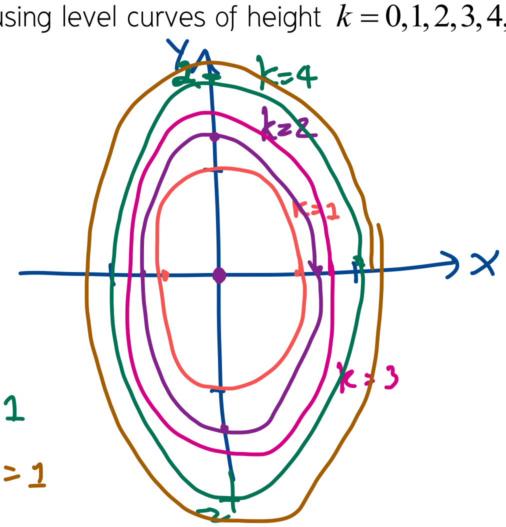
$k=1$; $1 = 4x^2 + y^2 \Leftrightarrow \frac{x^2}{\frac{1}{4}} + \frac{y^2}{1} = 1$

$k=2$; $2 = 4x^2 + y^2 \Leftrightarrow \frac{x^2}{\frac{1}{2}} + \frac{y^2}{2} = 1$

$k=3$; $3 = 4x^2 + y^2 \Leftrightarrow \frac{x^2}{\frac{3}{4}} + \frac{y^2}{3} = 1$

$k=4$; $4 = 4x^2 + y^2 \Leftrightarrow x^2 + \frac{y^2}{4} = 1$

$k=5$; $5 = 4x^2 + y^2 \Leftrightarrow \frac{x^2}{\frac{5}{4}} + \frac{y^2}{5} = 1$



4. Level surfaces

Observe that the graph of $y = f(x)$ is a curve in 2–space, and the graph of $z = f(x, y)$ is a surface in 3–space, so the number of dimensions required for these graphs is one greater than the number of independent variables. Accordingly, there is no "direct" way to graph a function of three variables since four dimensions are required. However, if k is a constant, then the graph of the equation $f(x, y, z) = k$ will generally be a surface in 3–space (e.g., $x^2 + y^2 + z^2 = 1$ the graph of is a sphere), which we call the **level surface with constant** k. Some geometric insight into the behavior of the function f can sometimes be obtained by graphing these level surfaces for various values of k.

Example 6 Describe the level surfaces of

(a) $f(x, y, z) = x^2 + y^2 + z^2$

$k = x^2 + y^2 + z^2$

$k = 0$

$k = 1$

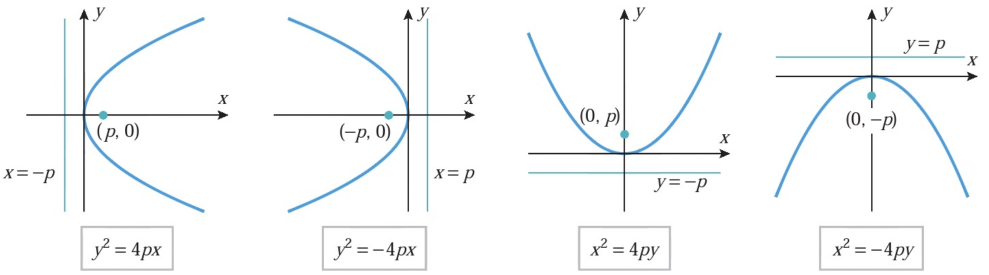
$k = 2$

(b) $f(x, y, z) = z^2 - x^2 - y^2$

ทบทวนภาคตัดกรวย

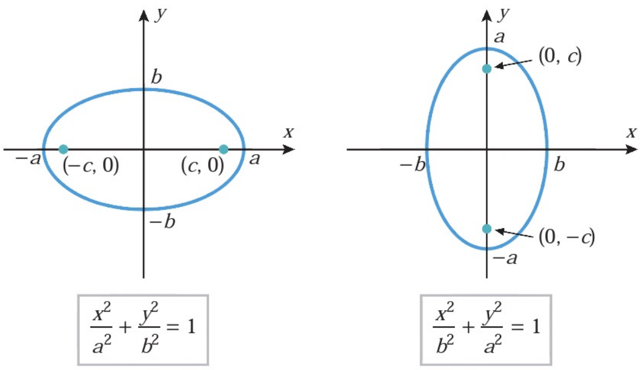
206112 QUICK REFERENCE

PARABOLAS IN STANDARD POSITION



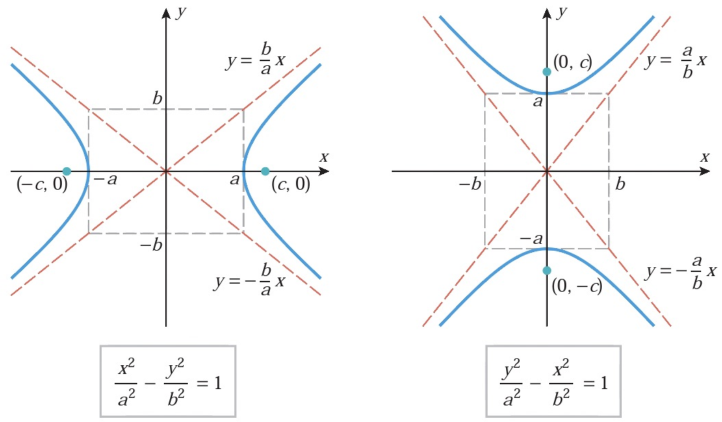
ELLIPSES IN STANDARD POSITION

$c = \sqrt{a^2 - b^2}$



HYPERBOLAS IN STANDARD POSITION

เงื่อนไข: $c = \sqrt{a^2 + b^2}$



ภาคตัดกรวยแบบเลื่อนขนาน

Parabolas with vertex (h, k) and axis parallel to x-axis

$$(y-k)^2 = 4p(x-h) \quad \text{[Opens right]}$$

$$(y-k)^2 = -4p(x-h) \quad \text{[Opens left]}$$

Parabolas with vertex (h, k) and axis parallel to y-axis

$$(x-h)^2 = 4p(y-k) \quad \text{[Opens up]}$$

$$(x-h)^2 = -4p(y-k) \quad \text{[Opens down]}$$

Ellipse with center (h, k) and major axis parallel to x-axis

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad [b < a]$$

Ellipse with center (h, k) and major axis parallel to y-axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad [b < a]$$

Hyperbola with center (h, k) and focal axis parallel to x-axis

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Hyperbola with center (h, k) and focal axis parallel to y-axis

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$