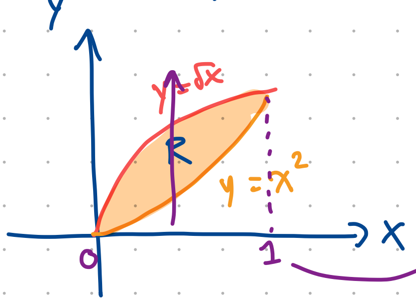
ทบทวน ① ให้ R เป็นบริเวณที่ปิดล้อมด้วยรูป จงเขียน $\iint_R f(x,y)\,dA$
(บฝน.3.1.2 ข้อ 10) ในรูปของการอินทิเกรตซ้ำ (Sheet ข้อ 9

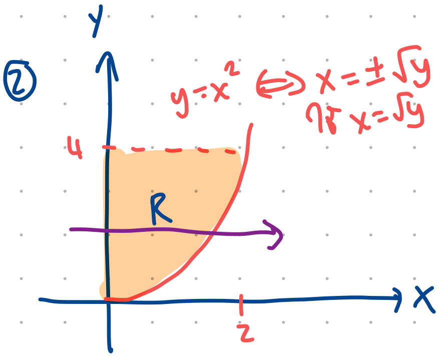


$$\int_0^1 \int_{x^2}^{\sqrt{x}} f(x,y)\,dy\,dx$$

หาจุดตัด

$$\sqrt{x}=x^2$$
$$x=x^4$$
$$x^4-x=0$$
$$x(x^3-1)=0$$
$$x(x-1)(x^2+x+1)=0 \Rightarrow x=0,1$$

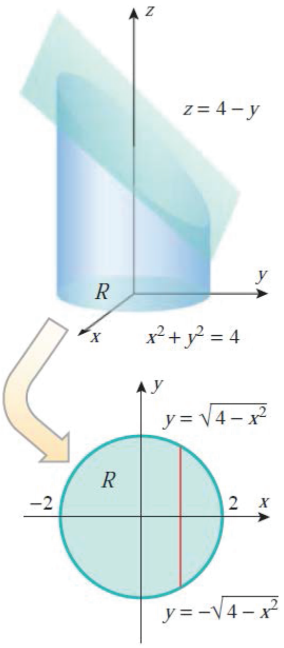
②



$y=x^2 \Leftrightarrow x=\pm\sqrt{y}$
ใช้ $x=\sqrt{y}$

$$\int_0^4 \int_0^{\sqrt{y}} f(x,y)\,dx\,dy = \int_0^2 \int_{x^2}^{4} f(x,y)\,dy\,dx$$

Example 10. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.



From: Calculus Early Transcendentals, 10th edition, Howard Anton, Irl C. Beven, Stephen Deavis, page 1013

Reversing the Order of Integration

Sometimes the evaluation of an iterated integral can be simplified by reversing the order of integration. The next example illustrates how this is done.

Example 11. Since there is no elementary antiderivative of e^{x^2}, the integral

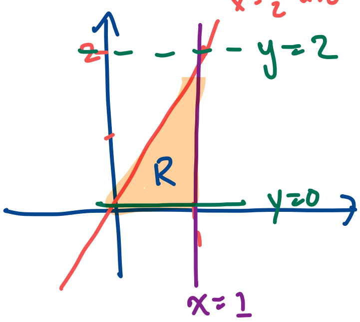
$$\int_0^2 \int_{x=y/2}^{x=1} e^{x^2}\, \underline{dx}\, dy \qquad \int e^{x^2} dx$$

cannot be evaluated by performing the x-integration rst. Evaluate this integral by express-ing it as an equivalent iterated integral with the order of integration reversed.

Solution.

$x = \frac{y}{2} \Rightarrow y = 2x$

$x = \frac{y}{2}$ หรือ $y = 2x$



$$\int_0^2 \int_{y/2}^{1} e^{x^2} dx\,dy = \int_0^1 \int_0^{2x} \underline{e^{x^2}}\, dy\,dx$$

$$= \int_0^1 y e^{x^2} \Big|_{y=0}^{y=2x} dx$$

$$= \int_0^1 2x e^{x^2} dx$$

ให้ $u = x^2$; $du = 2x\,dx$

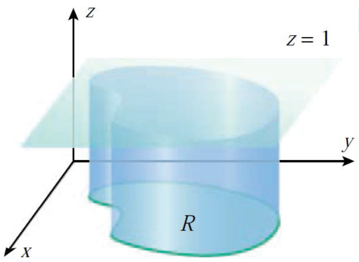
ขอบ: $x = 0$ ได้ $u = 0$

$x = 1$ ได้ $u = 1$

$$= \int_{u=0}^{u=1} e^u\, du = e^1 - e^0 = e - 1 \quad \#$$

Area calculated as a double integral

$$\text{area of } R = \iint_R 1\, dA = \iint_R dA$$



Cylinder with base R and height 1

หา พ.ท. ในระนาบ xy   เดิม $\iint_R f(x,y)\,dA$

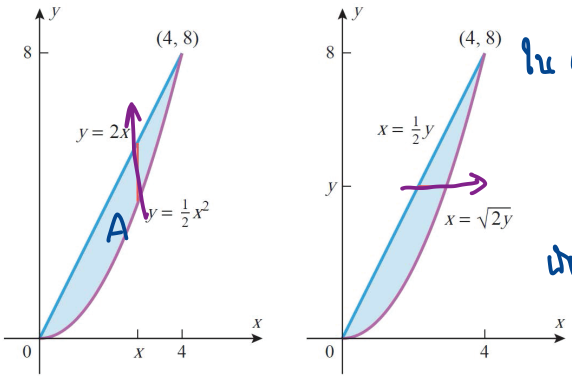
Set $f(x,y) = 1 \Leftrightarrow \iint_R f(x,y)\,dA = \iint_R dA$

$\therefore \iint_R dA$ คือ พ.ท. ของบริเวณ R

From: Calculus Early Transcendentals, 10th edition, Howard Anton, Irl C. Beven, Stephen Deavis, page 1014

Example 12. Use a double integral to nd the area of the region R enclosed between the parabola $y = \dfrac{x^2}{2}$ and the line $y = 2x$

ใน Cal 1 $A = \displaystyle\int_0^4 \left[2x - \tfrac{1}{2}x^2\right] dx$



ใน CAL 2 $A = \displaystyle\iint_R dA$

$$= \int_0^4 \int_{\frac{1}{2}x^2}^{2x} dy\,dx = \int_0^8 \int_{\frac{1}{2}y}^{\sqrt{2y}} dx\,dy$$

[illegible] $\displaystyle\int_0^4 \int_{\frac{1}{2}x^2}^{2x} dy\,dx = \int_0^4 y\Big|_{y=\frac{1}{2}x^2}^{2x} dx$

$$= \int_0^4 \left[2x - \tfrac{1}{2}x^2\right] dx$$

$$= \left[x^2 - \frac{x^3}{6}\right]_{x=0}^{x=4}$$

$$= 16 - \frac{64}{6} = \frac{32}{6} = \frac{16}{3} \quad \#$$

From: Calculus Early Transcendentals, 10th edition, Howard Anton, Irl C. Beven, Stephen Deavis, page 1014

EXERCISES 3.1.2

1-8 Evaluate the iterated integrals.

1. $\displaystyle\int_0^1 \int_{x^2}^{x} xy^2\,dy\,dx$

2. $\displaystyle\int_1^{3/2} \int_y^{3-y} y\,dx\,dy$

3. $\displaystyle\int_0^3 \int_0^{\sqrt{9-y^2}} y\,dx\,dy$

4. $\displaystyle\int_{1/4}^1 \int_{x^2}^{x} \sqrt{\frac{x}{y}}\,dy\,dx$

5. $\displaystyle\int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_0^{x^2} \sin\frac{y}{x}\,dy\,dx$

6. $\displaystyle\int_{-1}^1 \int_{-x^2}^{x^2} (x^2 - y)\,dy\,dx$

7. $\displaystyle\int_0^1 \int_0^{x} y\sqrt{x^2 - y^2}\,dy\,dx$

8. $\displaystyle\int_1^2 \int_0^{y^2} e^{x/y^2}\,dx\,dy$

REVIEW Midterm 2/2562

CH1 DEs Order 2

$$\boxed{ay''+by'+cy = G(x)}$$

- homogeneous $(G(x)=0)$
- non-homogeneous $(G(x)\neq 0)$

① <u>homogeneous</u> : $ay''+by'+cy=0 \iff ar^2+br+c=0$

- Initial value problem $y(0),\ y'(0)$
- Boundary value problem $y(a),\ y(b)$

① มี r_1, r_2 เป็นรากจริงที่แตกต่างกัน

$y(x)=c_1e^{r_1x}+c_2e^{r_2x}$

② มีรากซ้ำ r

$y(x)=c_1e^{rx}+c_2xe^{rx}$

② มีรากเป็นจำนวนเชิงซ้อน $r_1=\alpha+i\beta$, $r_2=\alpha-i\beta$

$y(x)=e^{\alpha x}[c_1\cos\beta x+c_2\sin\beta x]$

② <u>non-homogeneous</u> $ay''+by'+cy=G(x) \iff y(x)=y_c(x)+y_p(x)$

กำหนด $G(x)$ สมมติตามรูป $G(x)$

<u>Ex</u> $G(x)=x^2$ สมมติ $y_p(x)=Ax^2+Bx+C$

ต้องระวังไม่ให้ $y_p(x)$ ไปซ้ำกับ $y_c(x)$

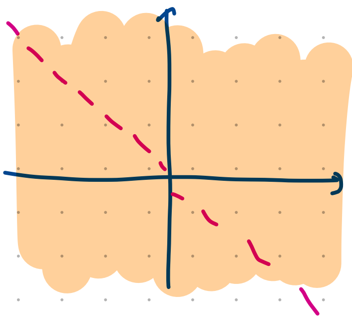
↳ หา สปส. $A, B, C, \ldots$

CH2 Function of Several Variables

① $f(x,y)=$ ______ <u>Domain</u> $=\{(x,y) \mid$ ______ $\}$

<u>Ex</u> $f(x,y)=\dfrac{1}{x+y}$; D_f หา $x+y\neq 0$

$y\neq -x$



$D_f=\{(x,y) \mid y\neq -x\}$

2. กราฟพื้นฐานในสามมิติ

- ระนาบ
- ทรงกลม
- ทรงกระบอก

} ร่างกราฟ โดยระนาบไปตัด xy , yz , xz

Qudric Surface → Ellipsoid → $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Hyperboloid 1 sheet → $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Hyperboloid 2 sheets → $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Elliptic cone → $z^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Elliptic Paraboloid → $z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

hyperbolic paraboloid → $z=\frac{y^2}{b^2}-\frac{x^2}{a^2}$

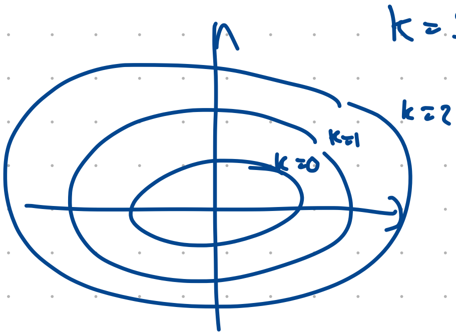
$\boxed{1}$ เลื่อนแกนขนาน

$\boxed{2}$ สะท้อน

3. Level curves / $f(x,y) = \boxed{}$ ⟺ $z=k$ เส้นโค้งตามระนาบ xy เมื่อตัดได้

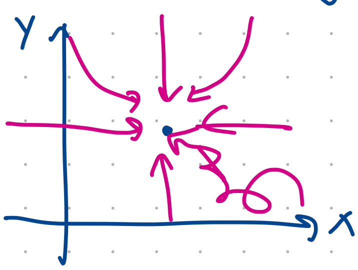
Contour map

$k=1,2,0,-1,-2$



Level surfaces

4. Limit + Continuity



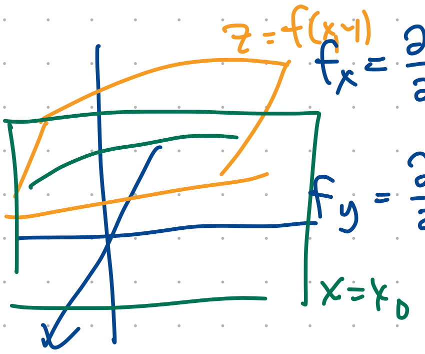
- ลิมิตตามเส้น → ลิมิต ของฟังก์ชันจะไม่มีค่า ถ้าเราหา เส้นสองเส้นที่แตกต่างกัน แล้วทำให้ค่าลิมิตตามเส้น ไม่เท่ากัน

ความต่อเนื่อง

1. $f(a,b)$ exists
2. $\lim_{(x,y)\to(a,b)} f(x,y)$ exist
3. $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

Ex $f(x) = \sin x$, $g(y) = y^2$ } $h(x,y) = f(x)\cdot g(y) = y^2 \sin x$

⑤ Partial Derivative $z=f(x,y)$



$$f_x = \frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$f_y = \frac{\partial z}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

- ใช้สูตร
- implicit

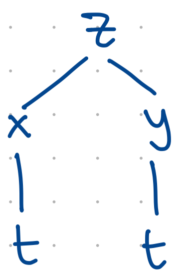
$w = f(x,y,z)$; f_x, f_y, f_z

④ higher-derivative

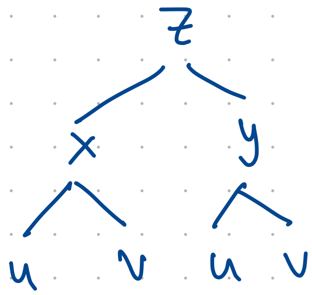
f_{xy} , $\frac{\partial^2 f}{\partial x \partial y}$

⑤ Chain rule

วาดแผนภาพเสมอ

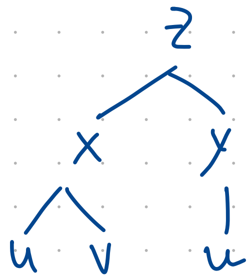


$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$



$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$



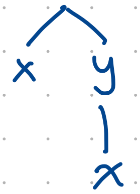
$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{dy}{du}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v}$$

⑥ Implicit diff

$f(x,y) = 0$; $\frac{dy}{dx} = -\frac{f_x}{f_y}$

$z = f(x,y)$



$F(x,y,z) = 0$; $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$

$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$

⑦ Differential ① $z = f(x,y)$; $dz = f_x(a,b)\,dx + f_y(a,b)\,dy$

② Local linear approximation

$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)dx + f_y(x_0,y_0)dy$

⑧ Extrema → Relative ext. → ① จุดวิกฤต $\left.\begin{array}{l} f_x = 0 \\ f_y = 0 \end{array}\right\}$ หา (x,y) ที่ทำให้ $f_x(x,y) = 0 = f_y(x,y)$

→ Absolute ext.

① หาจุดวิกฤต

② หาจุดบนขอบต่างๆ / จุดตามเส้นต่างๆ แล้วเอามาแข่งกัน

② ทดสอบจุดวิกฤต

$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$

$\boxed{D}$ @ (x_0,y_0)

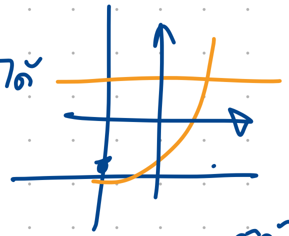
- $D > 0$
  - $f_{xx} > 0$
  - $f_{xx} < 0$
- $D < 0$
- $D = 0$

CH3 Multiple integrals (double Int) $\iint_R f(x,y)\,dA$

① R ขอบเขตเป็น □ Fubini's Thm

② R ขอบเขตเป็นรูปใดๆ

① วาดบริเวณได้



$$V = \iint_R f(x,y)\,dA = \int\int f(x,y)\,dy\,dx$$

$$= \int\int f(x,y)\,dx\,dx$$

② สลับลำดับการอินทิเกรตได้

③ $A = \iint_R 1\,dA$

Midterm F. 21 Feb, 15.30-16.30

RB5105