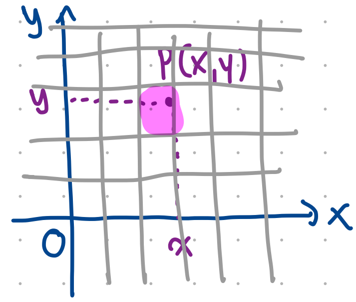
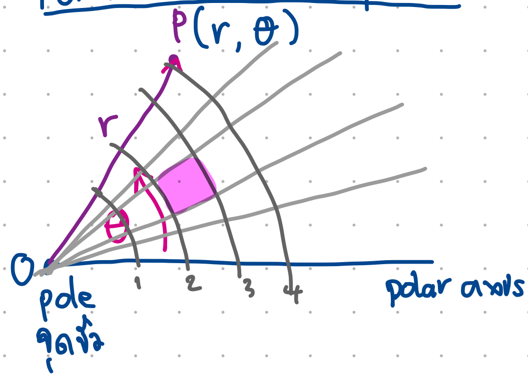
Rectangular coordinate

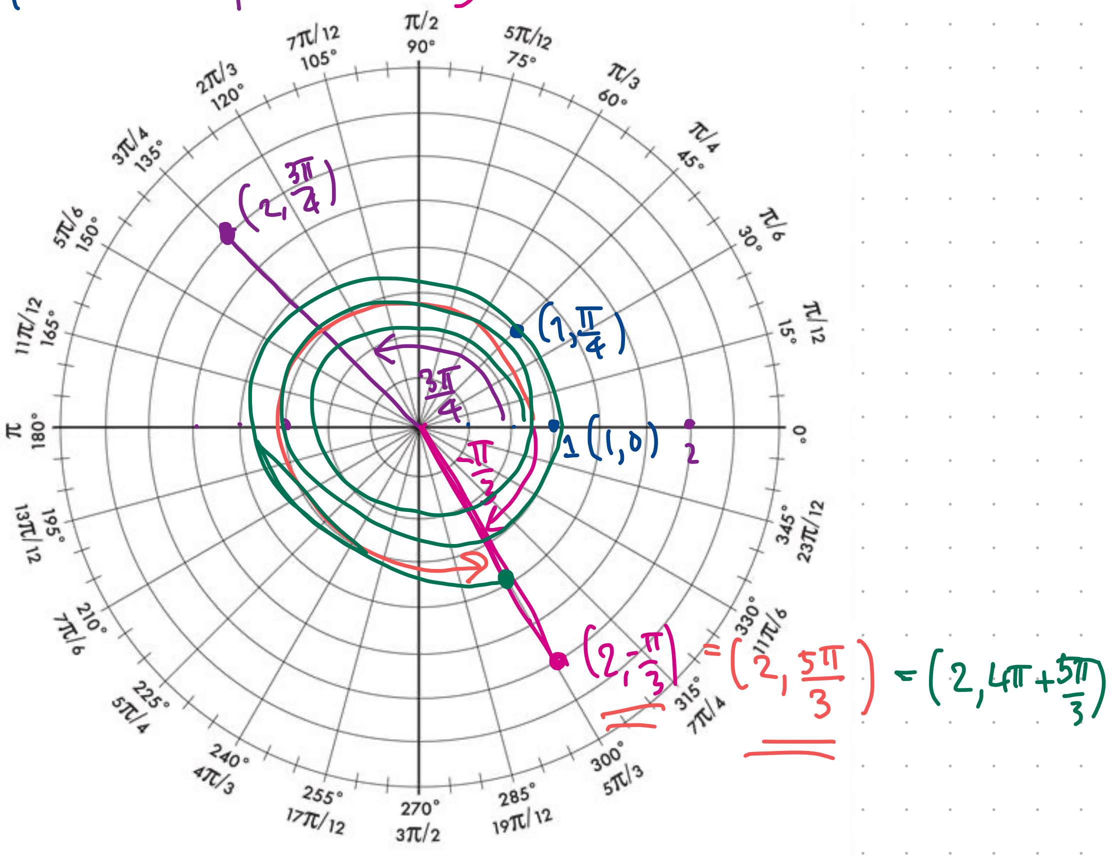


Polar Coordinate system

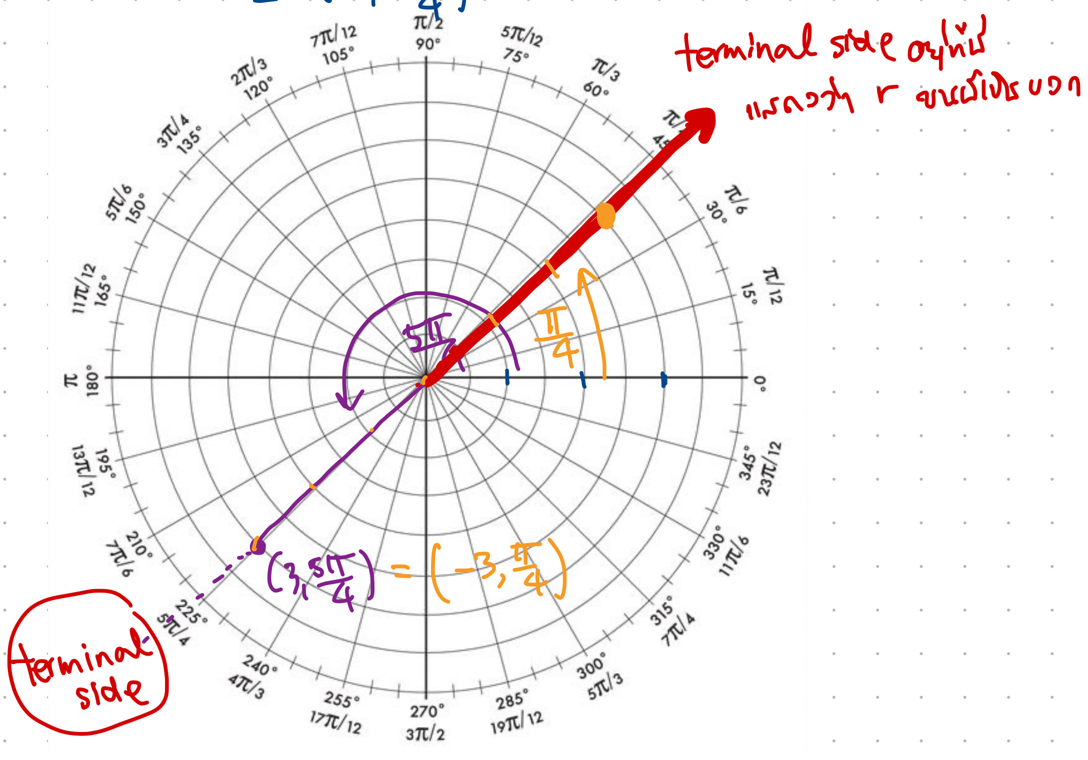


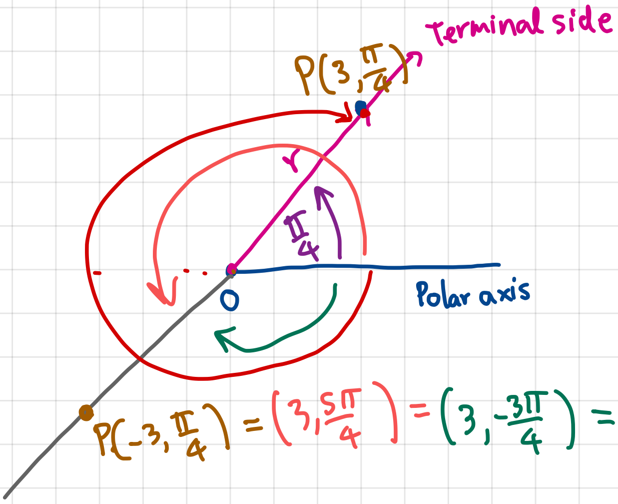
basis

Ex $(1, \frac{\pi}{4})$, $(2, \frac{3\pi}{4})$, $(2, -\frac{\pi}{3})$



ค่า r สามารถเป็นลบได้ Ex $(3, \frac{5\pi}{4})$





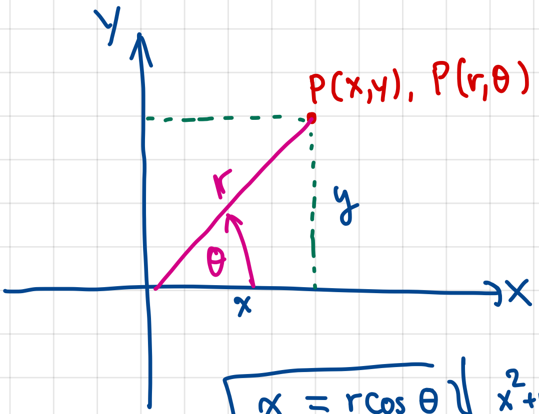
$P(-3,\frac{\pi}{4}) = (3,\frac{5\pi}{4}) = (3,-\frac{3\pi}{4}) = (-3,-\frac{7\pi}{4})$

* มุมเป็นลบ → วัดมุมตามเข็มนาฬิกา

* ค่า r เป็นบวก ถ้าวัดไปตาม terminal side

ค่า r เป็นลบ ถ้าวัดไป ตรงข้าม terminal side.

ความสัมพันธ์ระหว่างระบบพิกัดฉาก กับระบบพิกัดเชิงขั้ว



$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$x^2+y^2 = (r\cos\theta)^2+(r\sin\theta)^2$

$\therefore x^2+y^2 = r^2(\cos^2\theta+\sin^2\theta)$

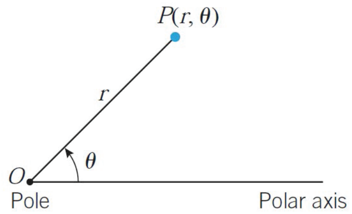
$\therefore \boxed{x^2+y^2=r^2}$

$\therefore \boxed{\tan\theta = \frac{y}{x}}$

3.2 Polar Coordinates and Graphs

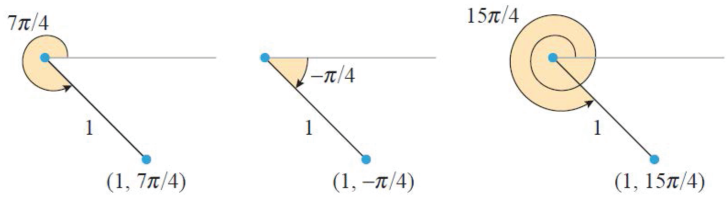
Polar Coordinate Systems

A *polar coordinate system* in a plane consists of a xed point O, called the *pole* (or *origin*), and a ray emanating from the pole, called the *polar axis.* In such a coordinate system, we can associate with each point P in the plane a pair of polar coordinates (r, θ), where r is the distance from P to the pole and θ is an angle from the polar axis to the ray OP. The number r is called the *radial coordinate* of P and the number θ the *angular coordinate* (or *polar angle*)of P.



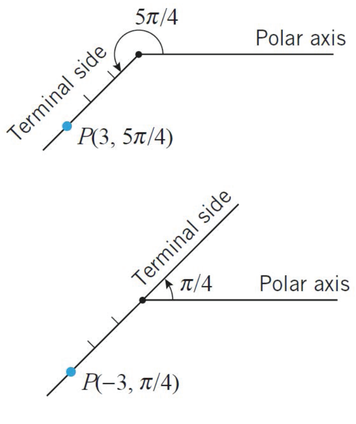
Remarks.

1. The polar coordinates of a point are not unique.



2. In general, if a point P has polar coordinates (r, θ), then $(r, \theta + 2n\pi)$ and $(r, \theta - 2n\pi)$ are also polar coordinates of P for any nonnegative integer n. Thus, every point has infinitely many pairs of polar coordinates.

3. In general, the terminal side of the angle $\theta + \pi$ is the extension of the terminal side of θ, so we define negative radial coordinates by agreeing that $(-r, \theta)$ and $(r, \theta + \pi)$ are polar coordinates of the same point.

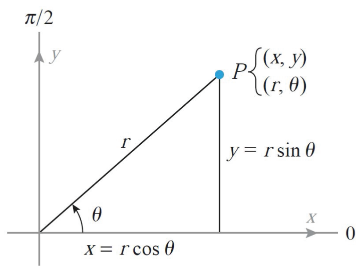


From: Calculus Early Transcendentals, 10th edition, Howard Anton, Irl C. Beven, Stephen Deavis, page 706

Relationship between Polar and Rectangular coordinates

As suggested by below figure, these coordinates are related by the equations

$$x = r\cos\theta, \quad y = r\sin\theta$$
$$r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}.$$



From: Calculus Early Transcendentals, 10th edition, Howard Anton, Irl C. Beven, Stephen Deavis, page 707

Example 1. Find the rectangular coordinates of the point P whose polar coordinates are $(r, \theta) = (6, 2\pi/3)$. $\in Q_2$

เชิงขั้ว → พิกัดฉาก

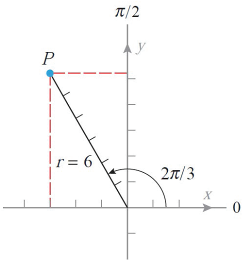
$x = r\cos\theta$

$y = r\sin\theta$

"Correspondence acute angle"

sin All
tan cos

[illegible]



$\therefore \; x = 6\cos\left(\frac{2\pi}{3}\right) = 6\left(-\cos\left(\frac{\pi}{3}\right)\right) = 6\cdot\left(-\frac{1}{2}\right) = -3$

$y = 6\sin\left(\frac{2\pi}{3}\right) = 6\cdot\sin\left(\frac{\pi}{3}\right) = 6\cdot\frac{\sqrt{3}}{2} = 3\sqrt{3}$

∴ จุดในระบบพิกัดฉากที่สอดคล้องคือ $(-3, 3\sqrt{3})$

Example 2. Find the rectangular coordinates of the point P whose rectangular coordinates are $(-2, -2\sqrt{3})$. $\in Q_3$

All possible cases: ① $r > 0$; $0 \le \theta < 2\pi$ ② $r < 0$; $0 \le \theta < 2\pi$ ③ $r > 0, -2\pi < \theta \le 0$ ④ $r < 0, -2\pi < \theta < 0$

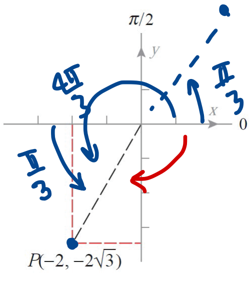
$x = r\cos\theta$

$y = r\sin\theta$

$\Updownarrow$

$x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x}$



รู้ $x, y \Rightarrow$ หา r, θ

$r^2 = (-2)^2 + (-2\sqrt{3})^2$

$r^2 = 16$

$r = \pm 4$

θ; $\tan\theta = \frac{-2\sqrt{3}}{-2} = \sqrt{3} \Leftrightarrow \theta = \frac{\pi}{3}, \frac{4\pi}{3}$

$P(-2, -2\sqrt{3})$ (ฉาก) $= P\left(4, \frac{4\pi}{3}\right)$ (เชิงขั้ว)

$= P\left(-4, \frac{\pi}{3}\right)$

$= P\left(4, -\frac{2\pi}{3}\right)$

$= P\left(-4, -\frac{5\pi}{3}\right)$

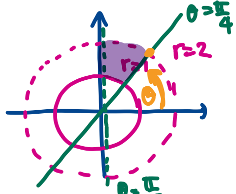
Graphs in Polar Coordinates

We will now consider the problem of graphing equations in r and θ, where θ is assumed to be measured in radians. In a rectangular coordinate system the graph of an equation in x and y consists of all points whose coordinates (x, y) satisfy the equation. However, in a polar coordinate system, points have innitely many different pairs of polar coordinates, so that a given point may have some polar coordinates that satisfy an equation and others that do not.

Given an equation in r and θ, we dene its graph in polar coordinates to consist of all points with at least one pair of coordinates (r, θ) that satisfy the equation.

$x = r\cos\theta$
$y = r\sin\theta$ $\Leftrightarrow$ $x^2 + y^2 = r^2$
$\tan\theta = \frac{y}{x}$

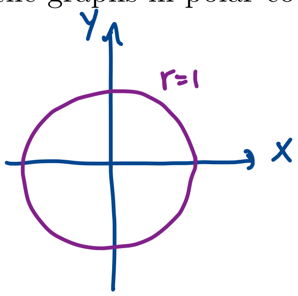
Ex จงเขียนกราฟบริเวณซึ่ง $1 \le r < 2$ และ $\frac{\pi}{4} \le \theta < \frac{\pi}{2}$



Example 3. Sketch the graphs in polar coordinates

(a) $r = 1$ วงกลม

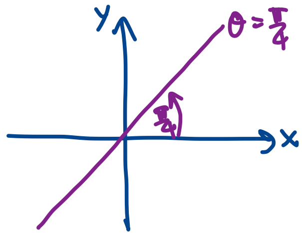
$x^2 + y^2 = r^2 = 1$



(b) $\theta = \frac{\pi}{4}$ เส้นตรง

$\tan\frac{\pi}{4} = \frac{y}{x} \Leftrightarrow 1 = \frac{y}{x}$

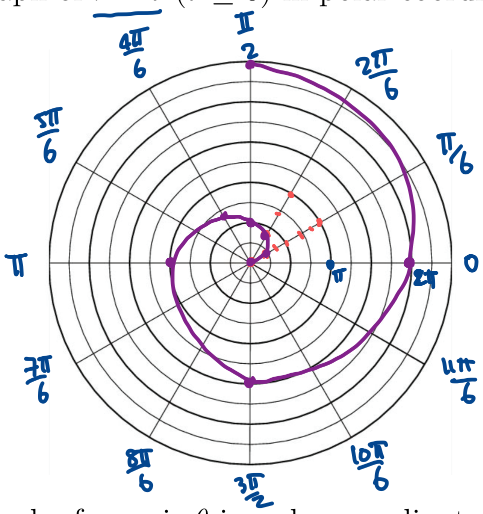
$\therefore y = x$



Equations $r = f(\theta)$ that express r as a function of θ are especially important. One way to graph such an equation is to choose some typical values of θ, calculate the corresponding values of r, and then plot the resulting pairs (r, θ) in a polar coordinate system. The next two examples illustrate this process.

Example 4. Sketch the graph of $r = \theta$ ($\theta \ge 0$) in polar coordinates by plotting points.

θ	r
0	0
$\frac{\pi}{6}$	$\frac{\pi}{6}$
$\frac{\pi}{3}$	$\frac{\pi}{3}$
$\frac{\pi}{2}$	$\frac{\pi}{2}$
π	π
$\frac{3\pi}{2}$	$\frac{3\pi}{2}$
2π	2π



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Example 5. Sketch the graph of $r = \sin\theta$ in polar coordinates by plotting points.

θ (RADIANS)	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$r = \sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0
(r, θ)	$(0, 0)$	$\left(\frac{1}{2}, \frac{\pi}{6}\right)$	$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right)$	$\left(1, \frac{\pi}{2}\right)$	$\left(\frac{\sqrt{3}}{2}, \frac{2\pi}{3}\right)$	$\left(\frac{1}{2}, \frac{5\pi}{6}\right)$	$(0, \pi)$	$\left(-\frac{1}{2}, \frac{7\pi}{6}\right)$	$\left(-\frac{\sqrt{3}}{2}, \frac{4\pi}{3}\right)$	$\left(-1, \frac{3\pi}{2}\right)$	$\left(-\frac{\sqrt{3}}{2}, \frac{5\pi}{3}\right)$	$\left(-\frac{1}{2}, \frac{11\pi}{6}\right)$	$(0, 2\pi)$

$r = \sin\theta \Leftrightarrow r^2 = r\sin\theta = y$

$x^2 + y^2 = r^2$

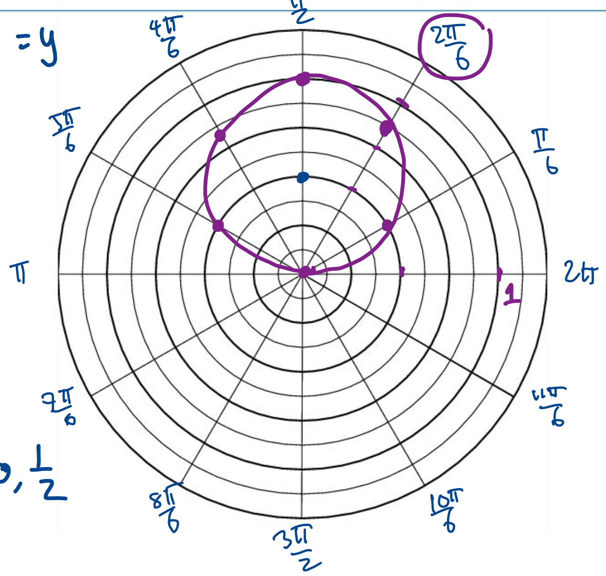
$\therefore x^2 + y^2 = y$

$x^2 + y^2 - y = 0$

$x^2 + (y^2 - y + \frac{1}{4}) = \frac{1}{4}$

$x^2 + (y - \frac{1}{2})^2 = (\frac{1}{2})^2$

วงกลมรัศมี $\frac{1}{2}$, จุดศูนย์กลางอยู่ที่ $(0, \frac{1}{2})$



$r = \cos\theta$